ℓ_0TV: A New Method for Image Restoration in the Presence of Impulse Noise

Ganzhao Yuan[1], Bernard Ghanem[2]

[1]South China University of Technology (SCUT), P.R. China. [2]King Abdullah University of Science and Technology (KAUST), Saudi Arabia.

Total Variation (TV) is an effective and popular prior model in the field of regularization-based image processing. This paper focuses on TV for image restoration in the presence of impulse noise. This type of noise frequently arises in data acquisition and transmission due to many reasons, e.g. a faulty sensor or analog-to-digital converter errors. Removing this noise is an important task in image restoration.

Table 1: Data Fidelity Models

Data Fidelity Function	Noise
$\ell_2(\mathbf{Ku},\mathbf{b}) = \|\mathbf{Ku}-\mathbf{b}\|_2^2$	additive Gaussian
$\ell_1(\mathbf{Ku},\mathbf{b}) = \|\mathbf{Ku}-\mathbf{b}\|_1$	additive Laplace
$\ell_\infty(\mathbf{Ku},\mathbf{b}) = \|\mathbf{Ku}-\mathbf{b}\|_\infty$	additive uniform
$\ell_p(\mathbf{Ku},\mathbf{b}) = \langle \mathbf{Ku}-\mathbf{b}\odot\log(\mathbf{Ku}), \mathbf{1}\rangle$	multiplicative Poisson
$\ell_g(\mathbf{Ku},\mathbf{b}) = \langle \log(\mathbf{Ku})+\mathbf{b}\odot\frac{1}{\mathbf{Ku}}, \mathbf{1}\rangle$	multiplicative Gamma
$\ell_{02}(\mathbf{Ku},\mathbf{b}) = \|\mathbf{Ku}-\mathbf{b}+\mathbf{z}\|_2^2,\ s.t.\ \|\mathbf{z}\|_0 \le k$	mixed Gaussian impulse [4]
$\ell_0(\mathbf{Ku},\mathbf{b}) = \|\mathbf{Ku}-\mathbf{b}\|_0$	add./mult. impulse, **[this paper]**

Contributions: (i) ℓ_0-norm data fidelity is proposed to address the TV-based image restoration problem. Compared with existing models, such as ℓ_{02}-norm data fidelity, our model is particularly suitable for image restoration in the presence of impulse noise. The fidelity function penalizes the difference between $\mathbf{Ku}$ and $\mathbf{b}$ by using different norms/divergences. Its form depends on the assumed distribution of the noise model. For comparisons, we list some typical noise models and their corresponding fidelity terms in Table 1. Generally speaking, we consider the following $\ell_0 TV$ model:

$$\min_{\mathbf{0}\le\mathbf{u}\le\mathbf{1}} \|\mathbf{Ku}-\mathbf{b}\|_0 + \lambda TV(\mathbf{u}) \tag{1}$$

(ii) To deal with the resulting NP-hard ℓ_0 norm optimization problem, we propose a proximal ADMM method to solve an equivalent MPEC (Mathematical Program with Equilibrium Constraints) form of the problem in Eq (1). Specifically, using the variational characterization of ℓ_0-norm [1, 2]

$$\|\mathbf{w}\|_0 = \min_{\mathbf{0}\le\mathbf{v}\le\mathbf{1}} \langle \mathbf{1}, \mathbf{1}-\mathbf{v}\rangle,\ s.t.\ \mathbf{v}\odot|\mathbf{w}| = \mathbf{0},$$

we reformulate Eq (1) as the following equivalent MPEC problem:

$$\min_{\mathbf{0}\le\mathbf{u},\mathbf{v}\le\mathbf{1}} \langle \mathbf{1}, \mathbf{1}-\mathbf{v}\rangle + \lambda TV(\mathbf{u}), \quad \text{s.t.} \quad \mathbf{v}\odot|\mathbf{o}\odot(\mathbf{Ku}-\mathbf{b})| = \mathbf{0} \tag{2}$$

Problem (1) is equivalent to problem (2) in the sense that if $\mathbf{u}^*$ is a global optimal solution of Eq (1), then $(\mathbf{u}^*, \mathbf{1}-\text{sign}(|\mathbf{Ku}^*-\mathbf{b}|))$ is globally optimal to Eq (2). Conversely, if $(\mathbf{u}^*,\mathbf{v}^*)$ is a global optimal solution of Eq (2), then $\mathbf{u}^*$ is globally optimal to Eq (1). We argue that, from a practical perspective, improved solutions to Eq (1) can be obtained by reformulating the ℓ_0-norm in terms of complementarity constraints. We develop a proximal ADMM algorithm to solve the non-smooth non-convex problem in Eq (2) and prove that it is convergent to a KKT point under mild conditions.

Experiments: We provide empirical validation for our proposed $\ell_0 TV$-PADMM method by conducting extensive image denoising and deblurring experiments. First, we verify the convergence property of our $\ell_0 TV$-PADMM method by considering the 'cameraman' image in Figure 1. Clearly, the corrupting noise is being effectively removed throughout the optimization process. Secondly, we present an image restoration example on the well-known 'barbara' image using our proposed method in Figure 2. The recovery quality of the proposed method appears to be very satisfactory. Finally, we compare the performance of existing state-of-the-art methods on general denoising problems in Table 2. We make two important observations. (i) $\ell_0 TV$ model is more effective than Yan's $\ell_{02}TV$ model ($\ell_{02}TV$-AOP) [4] for reconstructing images corrupted with impulse noise. (ii) Our proposed proximal ADMM solver is more effective than the penalty decomposition algorithm ($\ell_0 TV$-PDA) [3] used for solving the ℓ_0TV problem, especially when the noise level is large. We attribute this result to the "lifting" technique that is used in our optimization algorithm.

Implementation: For the purpose of reproducibility, we provide our MATLAB code at: `http://yuanganzhao.weebly.com/`.

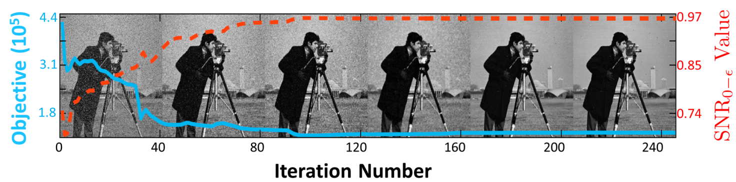

Figure 1: Asymptotic behavior for optimizing (1) on image denoising problems. We plot the value of the objective function (in solid blue line) and the SNR value (in dashed red line) against the number of iterations, as well as how the noise has been removed at different stage of the process (1, 10, 20, 40, 80, 160).

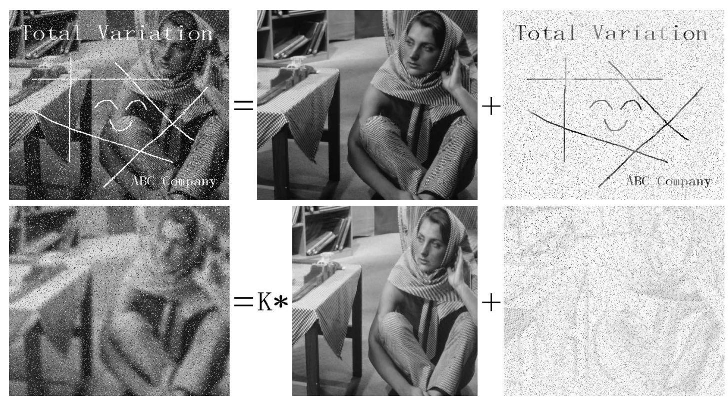

Figure 2: An example of image recovery result. Left column: corrupted image. Middle column: recovered image. Right column: complement of absolute residual between the corrupted image and recovered image.

Table 2: General Denoising Problems. The results separated by '/' are SNR_0, SNR_1 and SNR_2, respectively.

Img. \ Alg.	$\ell_1 TV$-SBM	TSM	$\ell_{02}TV$-AOP	$\ell_0 TV$-PDA	$\ell_0 TV$-$PADMM$
Random-Value Impulse Noise					
pirate+10%	0.93/10.06/15.58	0.96/13.26/14.26	**0.97**/13.26/17.13	**0.97**/**15.66**/**18.60**	**0.97**/15.46/17.78
pirate+30%	0.88/8.19/12.78	0.85/6.43/8.82	**0.93**/9.36/13.87	**0.93**/**11.46**/**14.88**	**0.93**/11.00/13.12
pirate+50%	0.65/4.69/7.27	0.67/3.16/4.92	0.83/6.95/10.28	0.87/8.64/**11.83**	**0.89**/**8.70**/10.60
pirate+70%	0.42/2.05/2.93	0.46/1.48/2.02	0.55/2.86/3.85	0.62/4.02/5.61	**0.82**/**6.74**/**8.54**
pirate+90%	0.26/0.36/0.12	0.26/0.21/-0.14	0.28/0.46/0.25	0.31/0.74/0.66	**0.51**/**2.26**/**2.40**
Salt-and-Pepper Impulse Noise					
pirate+10%	0.94/10.18/15.69	0.98/17.54/22.44	0.98/17.53/22.43	**0.99**/19.58/25.95	**0.99**/**19.97**/**26.63**
pirate+30%	0.90/8.66/12.90	0.97/13.80/19.47	0.97/13.76/19.39	**0.98**/14.23/19.98	**0.98**/**14.66**/**20.69**
pirate+50%	0.80/6.43/8.96	**0.96**/11.62/17.05	0.95/11.56/16.94	0.95/11.34/16.57	**0.96**/**11.87**/**17.36**
pirate+70%	0.58/3.21/5.49	**0.92**/9.48/14.10	**0.92**/9.46/14.07	0.89/8.78/13.22	**0.92**/**9.56**/**14.20**
pirate+90%	0.29/1.02/1.78	0.80/6.50/9.64	0.80/6.47/9.58	0.55/3.87/6.39	**0.81**/**6.60**/**9.72**

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This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.