Sparse Representation Classification with Manifold Constraints Transfer

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The fact that image data samples lie on a manifold has been successfully exploited in many learning and inference problems. In this paper we leverage the specific structure of data in order to improve recognition accuracy. In particular we propose a novel framework that allows to embed manifold priors into sparse representation-based classification (SRC) approaches [1].

SRC based methods define the problem of recognition as the optimization of a matrix Z of size $N \times F$ which describes the input data matrix H_r of size $M \times F$ given a dictionary H_d of size $M \times N$ where N and F are respectively the training and test samples while M represents the feature dimensionality. This is formalized as:

$$\begin{aligned} \underset{Z}{\text{minimize}} \quad & \|Z\|_1 \\ \text{subject to} \quad & H_r = H_d Z. \end{aligned} \tag{P1}$$

Our intuition is that manifold constraints can be transferred from the data to the optimized variables if these are linearly correlated. Using this new insight, we define an efficient alternating direction method of multipliers (ADMM) that can consistently integrate the manifold constraints during the optimization process. This is based on the fact that we can recast the problem as the projection over the manifold via a linear embedding method based on the Geodesic distance.

Figure 1 shows a scheme of our approach. At first, in the **data generation** step, given a test sample and the training samples, we augment the data by simulating geometrical transformations, e.g. rotation and translation, or by choosing k nearest neighbors from the training set. the **Manifold embedding** step process the augmented data and it finds an embedding to the manifold using Linear Local Embedding (LLE). Then we proved that, if testing and training data are linearly correlated through the assignment matrix Z, it is possible to transfer the manifold constraints to the latter matrix (**MCT proof** stage). This result simplifies the inference mechanism of the algorithm and it provides an efficient implementation through ADMM which we employed in the last step, called **optimization on** Z.

From an optimization point of view, we can write the original SRC problem (P1) to include manifold constraints on the data such that:

$$\begin{aligned} \underset{Z,\widehat{H}_r}{\text{minimize}} \quad & \mathcal{F}(Z,\widehat{H}_r) \\ \text{subject to} \quad & H_r = \widehat{H}_r \\ & \widehat{H}_r \in \mathcal{M}, \end{aligned} \tag{P2}$$

Then, we formalize the manifold using the notation of [3] by introducing a neighbour-preserving embedding which we use to find an estimate on a sub-manifold (a subset of a manifold). Such formalization is similar to the one of [2, 3] that first calculates the weights in the process of dimension reduction by LLE [4].

In particular, the embedding generated by [2, 3] is exactly based on a sub-manifold given by a small set of samples. Therefore, instead of finding a whole manifold (LLE) to the data, we alternatively construct a sub-manifold. We just consider the Geodesic distance information as in [2] contained in a sub-manifold to find a projection based on its "true" neighbours, and thus avoiding the use of perturbations of samples far from the input data.

Let $\mathcal{M}$ be the sample set representing a manifold and let m' be the embedding of $\mathcal{M}$ via a mapping function $\Phi(\cdot)$.

Definition 1 *The map function $\Phi : m' \to \mathcal{M}$ in the neighbour-preserving embedding method based on the Geodesic distance is defined as follows: $\Phi(m',\mathcal{M}) = \sum_{j=1}^{K}(1-W_j)\mathcal{M}_j$ where W_j is the Geodesic distance of the sample m' and the j^{th} sample in a sub-manifold set.*

This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.

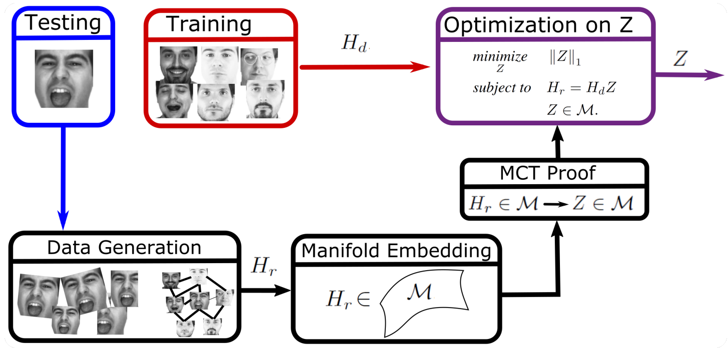

Figure 1: A scheme of the MCT approach.

Given the solution presented to (P2), we argue that the original objective is still complex to optimize because of the cloned variables involved. However by considering the relationship between the optimized variable and the input data from a manifold, we can further relax the minimization objective to become extremely efficient. Our solution guarantees that a new optimization method can solve (P2) by taking advantage of the linear relation between Z and the involved data H_r and $\widehat{H_r} \in \mathcal{M}$.

Given the previous problem, a practical solution can be obtained with the ADMM algorithm where the optimization with manifold constraints can be solved with a simpler projection to the manifold (as similarly done in [5]). We then show that the minimized objective function can be reformulated as:

$$\begin{aligned} \underset{Z}{\text{minimize}} \quad & \|Z\|_1 \\ \text{subject to} \quad & H_r = H_d Z \\ & Z \in \mathcal{M}. \end{aligned} \tag{P3}$$

where the last problem provides the same solution of (P2) but it is characterized by a simpler cost function given by

$$L_\sigma(Z) = ||Z||_1 + \sigma \sum_{i=1}^{F} \left\| \mathbf{z}_{r,i} - \mathbf{z}'_i \right\|_1 , \tag{1}$$

where Z' is the variable cloned from Z, for which $Z' \in \mathcal{M}$. In this way we detach the optimization of the l_1 norm cost from the manifold constraints that are affecting the cloned variable only. The manifold is defined again as a linear embedding as shown in Definition 1. Moving from (P2) to (P3), we can observe that the manifold constraint can be transferred from the data to the variable in the case of linear correlation, e.g., $H_r = H_d Z$. Thus for each sample from a given class (manifold), one can optimize the above objective separately just based on a initialized manifold sample space. This represents the main theoretical contribution of MCT.

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