## **Riemannian Coding and Dictionary Learning: Kernels to the Rescue**

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While sparse coding on non-flat Riemannian manifolds has recently become increasingly popular, existing solutions either are dedicated to specific manifolds, or rely on optimization problems that are difficult to solve, especially when it comes to dictionary learning. In this paper, we propose to make use of kernels to perform coding and dictionary learning on Riemannian manifolds. To this end, we introduce a general Riemannian coding framework with its kernel-based counterpart. This lets us (i) generalize beyond the special case of sparse coding; (ii) introduce efficient solutions to two coding schemes; (iii) learn the kernel parameters; (iv) perform unsupervised and supervised dictionary learning in a much simpler manner than previous Riemannian coding approaches.

More specifically, let  $\mathcal{D} = \{d_i\}_{i=1}^N$ ,  $d_i \in \mathcal{M}$ , be a dictionary on a Riemannian manifold  $\mathcal{M}$ , and  $x \in \mathcal{M}$  be a query point on the manifold. We define a general Riemannian coding formulation as

$$\min_{\alpha} \delta^2(x, \biguplus_{j=1}^N \alpha_j d_j) + \lambda \gamma(\alpha; x, \mathcal{D})$$
(1)  
s.t.  $\alpha \in \mathcal{C}$ ,

where  $\delta : \mathcal{M} \times \mathcal{M} \to \mathbb{R}^+$  is a metric on  $\mathcal{M}$ ,  $\alpha \in \mathbb{R}^N$  is the vector of Riemannian codes,  $\gamma$  is a prior on the codes  $\alpha$  and C is a set of constraints on  $\alpha$ . Moreover,  $\bigcup : \mathcal{M} \times \cdots \times \mathcal{M} \times \mathbb{R} \times \mathbb{R} \cdots \times \mathbb{R} \to \mathcal{M}$  is an operator that combines multiple dictionary atoms  $\{d_j \in \mathcal{M}\}$  with weights  $\{\alpha_j\}$  and generates a point  $\hat{x}$  on  $\mathcal{M}$ . This general formulation encapsulates intrinsic sparse coding [2, 5], but also lets us derive and intrinsic version of Locality-constrained Linear Coding [10]. Such intrinsic formulations, however, depend on the logarithm map, which may be highly nonlinear, or not even have an analytic solution.

To overcome these weaknesses and obtain a general formulation of Riemannian coding, we propose to perform coding in RKHS. This has the twofold advantage of yielding simple solutions to several popular coding techniques and of resulting in a potentially better representation than standard coding techniques due to the nonlinearity of the approach. To this end, let  $\phi : \mathcal{M} \to \mathcal{H}$  be a mapping to an RKHS induced by the kernel  $k(x,y) = \phi(x)^T \phi(y)$ . Coding in  $\mathcal{H}$  can then be formulated as

$$\min_{\alpha} \left\| \phi\left(x\right) - \sum_{j=1}^{N} \alpha_{j} \phi\left(d_{j}\right) \right\|_{2}^{2} + \lambda \gamma(\alpha; \phi\left(x\right), \phi\left(\mathcal{D}\right)) \\
\text{s.t.} \quad \alpha \in \mathcal{C}.$$
(2)

As shown in the paper, the reconstruction term in (2) can be kernelized. More importantly, after kernelization, this term remains quadratic, convex and similar to its counterpart in Euclidean space. This lets us derive efficient solutions to two coding schemes: kernel Sparse Coding (kSC) and kernel Locality Constrained Coding (kLCC).

In many cases, it is beneficial not only to compute the codes for a given dictionary, but also to optimize the dictionary to best suit the problem at hand. Given training data, and for fixed codes, we then show that, by relying on the *Representer theorem* [8], the dictionary update has an analytic form. Furthermore, we introduce an approach to supervised dictionary learning, which, given labeled data, jointly learns the dictionary and a classifier acting on the codes. The resulting supervised coding schemes are referred to as kSSC and kSLCC.

We demonstrate the effectiveness of our approach on three different types of non-flat manifolds, as well as illustrate its generality by also applying it to Euclidean space, which simply is a special type of Riemannian manifold. In particular, we evaluated our different techniques on two challenging classification datasets where the images are represented with region covariance descriptors (RCovDs) [9], which lie on SPD manifolds.

This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.

To this end, we considered two classification tasks: virus classification using the Kylberg dataset and material categorization using the KTH-TIPS2b dataset. Furthermore, to illustrate the effectiveness of our coding schemes in Euclidean space, we made use of the extended YALE-B dataset and of Caltech101. The results given in Tables 1 and 2 evidence the benefits of our kernel coding schemes over existing baselines, with kSLCC generally arising as the best performer. Additional results confirming this trend for Grassmann manifolds and for the shape manifold are provided in the paper and in supplementary material.



Method	Virus	KTH-TIPS2-b
CDL [11]	69.5%	76.3%
logEuc-SC	68.3%	67.8%
logEuc-LCC	72.3%	75.9%
kSC	78.5%	78.8%
kLCC	79.4%	79.8%
kSSC	81.7%	79.9%
kSLCC	82.0%	81 2%





Table 2: Coding in Euclidean space. Left panel. Samples from extended YALE-B [4]. Middle panel. Samples from Caltech101 [3]. Right panel. Recognition accuracies.

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