The Aperture Problem for Refractive Motion

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When viewed through a small aperture, a moving image provides incomplete information about the local motion. Only the component of motion along the local image gradient is constrained. In an essential part of optical flow algorithms, information must be aggregated from nearby image locations in order to estimate all components of motion. This limitation of local evidence for estimating optical flow is called "the aperture problem" [1].

We pose and solve a generalization of the aperture problem of moving refractive elements. We consider a common setup in air flow imaging or telescope observation: a camera is viewing a static background through an unknown refractive elements. While many motion estimation algorithms for refractive objects have been proposed [2, 3, 4], a fundamental question remains unanswered: what does the local image information tell us about the motion of refractive elements?

In this paper, we are going to discuss this problem. Formally, consider any planar and static background pattern $f(\mathbf{x})$ (we assume $f(\mathbf{x})$ is black-and-white pattern whose boundary is a straight line) and a camera observing it through a refractive layer. The observed sequence $g(\mathbf{x},t)$ at time t is

$$g(\mathbf{x},t) = f(\mathbf{x} - \mathbf{r}_0(\mathbf{x} - \mathbf{u}t)), \tag{1}$$

where $\mathbf{r}_0(\cdot)$ is the unknown warping field, and $\mathbf{u}$ is the motion of the refractive object to be recovered. Within a small spatiotemporal window, we assume that refractive motion $\mathbf{u}$ is constant. Then the question we are going to resolve is: whether it is possible to uniquely determine the motion vector $\mathbf{u}$ only from an observed sequence $g(\mathbf{x},t)$.

Main conclusion The ambiguity in refractive motion estimation is summarized as follows:

Case 1. When the background is totally plain, it is impossible to recover any information about refractive motion since no changes is observed in the captured sequence even if the reflective object is moving.

Case 2. When a first order structure (straight edge) is observed, no information about refractive motion can be recovered, as the same observed motion might due to either large motion of the refractive object and small refraction, or small motion and large refraction.

Case 3. When a second order structure (conic curve) is observed, we can recover the motion in only one direction. More specifically, when the refractive object is rotationally symmetric, we can calculate the component of the motion parallel to the background structure, but not the component that is perpendicular to it. Note that this is opposite to the traditional aperture problem, where only the component of the motion perpendicular to the background structure is recoverable. Moreover, if we fit a circle $a\mathbf{x}^\intercal\mathbf{x} + \mathbf{q}^\intercal\mathbf{x} + 1 = 0$ to the observed boundary at each time frame t, then refractive motion $\mathbf{u}$ should satisfy the simplified refractive flow equation:

$$-n_\perp^\intercal \frac{d\mathbf{q}/dt}{2a} = n_\perp^\intercal \mathbf{u}, \tag{2}$$

where $n_\perp$ is the direction of background structure. We also have the general refractive flow equation if the refractive object is not rotationally symmetric.

Experiment We verify our theory on sequences of both solid and fluid refractive objects. For solid refractive object, we took a sequence of a black and white background through a moving magnifying lens (Figure 1(a)). First, two lenses move perpendicularly to the background structure in different speeds (Figure 1(b)). The observed sequences with the aperture (red rectangle) are quite similar, which shows that it is hard to differentiate these two cases just from observation, and thus we cannot recover the horizontal component of the motion. Next, when a lens moves parallel to the background structure (Figure 1(c)), the observed boundary moves together with the lens, and thus we can recover the lens motion by tracking the observed boundary. This is consistent with our conclusion.

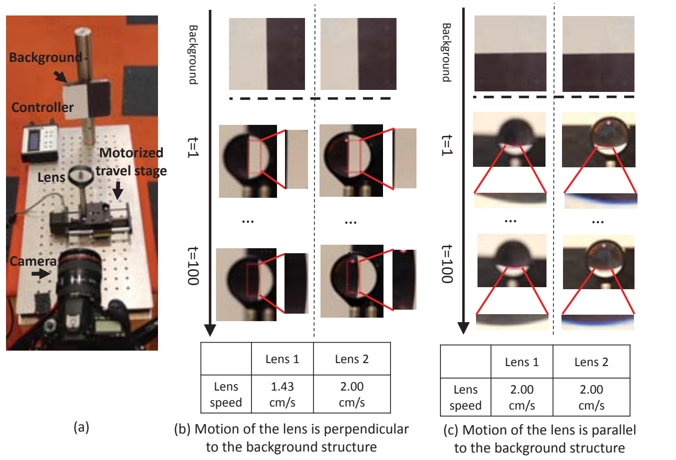

Figure 1: Experiments on magnifying lenses (refractive solid). (a) Experiment setup. (b) The motion of lens is perpendicular to the background structure. (c) The motion of lens is parallel to the background structure.

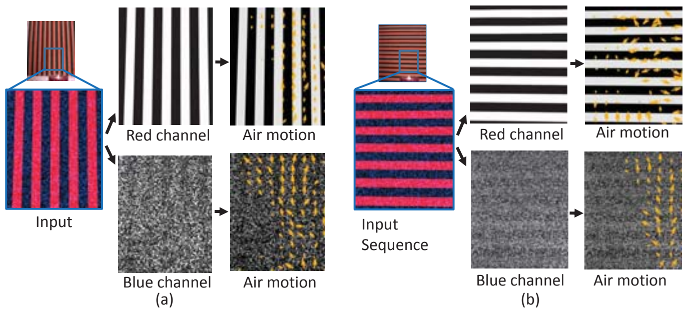

Figure 2: Experiments on hot air generated by a candle (refractive fluid). (a) The motion of hot air is parallel to the background structure. (b) The motion of hot air is perpendicular to the background structure.

For refractive fluid, we capture sequences through hot air generated by a burning candle (Figure 2). The background consists of patterns from two color channels. The blue channel is fully-textured, from which we can correctly recover the upward air motion (the second row of Figure 2), and we consider the recovered motion from this channel as the ground truth. Here we use the algorithm in [4] to calculate the air motion from input sequences. The red channel contains texture only in one direction (the first row in Figure 2). When the texture is perpendicular to the direction of air motion, the estimated motion from red channel is incorrect, and when the texture is parallel to the direction of air motion, the estimated motion from the red channel is roughly correct.

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This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.