Shape-from-Template in Flatland

Mathias Gallardo, Daniel Pizarro, Adrien Bartoli and Toby Collins
ALCoV-ISIT, UMR6284 CNRS / Universitś d'Auvergne, Clermont-Ferrand

Shape-from-Template (SfT) is the problem of inferring the shape of a deformable object as observed in an image using a shape template. We call 2DSfT the 'usual' instance of SfT where the shape is a surface embedded in 3D and the image a 2D projection. We introduce 1DSfT, a novel instance of SfT where the shape is a curve embedded in 2D and the image a 1D projection. We focus on isometric deformations, for which 2DSfT is a well-posed problem, and admits an analytical local solution [1] which may be used to initialize nonconvex refinement [2]. More precisely, 1DSfT consists in recovering a curve whose the length between two points is preserved by the deformation. 1DSfT appears like an easier version of 2DSfT since one dimension has been removed. However, our study reveals that 1DSfT is more complicated than seemingly.

This paper presents a theoretical study of 1DSfT for perspective projection and isometric deformations. We only consider planar scurves, but this study provide insights for the reconstruction of isometric non-planar curves, so-called curve-based 2DSfT, which has a wider set of applications. We show that the 1DSfT properties are different from the 2DSfT ones. A differential approach leads to the non-local solvability at any order, but also to the notion of critical points. These special points of the 2D curve provide some properties of the solution space. We propose an algorithm to compute 1DSfT solutions following our theoretical contribution. 1DSfT also gives a framework for testing new ideas in SfT, such as the angle-based parameterization.

Figure 1 illustrates the modeling of 1DSfT. The template $\mathcal{T} \subset \mathbb{R}$ is deformed smoothly into a curve $\mathcal{S} \subset \mathbb{R}^2$ by an embedding function $\varphi = (\varphi_x\ \varphi_y)^\top \in C^\infty(\mathcal{T}, \mathbb{R}^2)$. We note Π the perspective projection. The function $\eta \in C^\infty(\mathcal{T}, \mathbb{R})$ is the registration warp between the template and the image $\mathfrak{I} \subset \mathbb{R}$. We assume that $\mathcal{S}$ has no self-occlusions in $\mathfrak{I}$. Considering *reprojection* and *isometry* constraints, 1DSfT is equivalent to this problem:

$$\text{Find} \quad \varphi \in C^\infty(\mathcal{T}, \mathbb{R}^2) \quad s.t. \quad \begin{cases} \eta = \Pi \circ \varphi & \text{(reprojection)} \\ \|\varphi'\|_2^2 = 1 & \text{(isometry).} \end{cases} \tag{1}$$

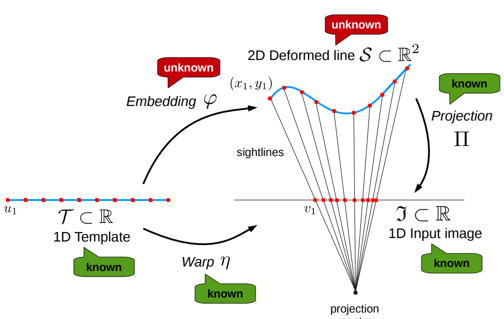

Figure 1: General modeling of 1DSfT, the problem of monocular template-based 2D reconstruction of a deformed curve.

We rewrite this formulation as a first order non-linear ODE:

$$(\varphi_y'\eta)^2 + 2\varphi_y\varphi_y'\eta\eta' + (\varphi_y\eta')^2 + (\varphi_y')^2 = 1. \tag{2}$$

To deal with equation (2), we use non-holonomic solutions [3] assuming that φ_y and φ_y' are independent variables. We prove that 1DSfT cannot be exactly solved locally. To study global solutions, we use a change of variable $\theta = \varphi_y\varepsilon$ with $\varepsilon = \sqrt{\eta^2+1}$ and we define the *critical points* as the points $u_c \in \mathcal{T}$ that cancel the transformed equation from equation (2):

$$\theta' = \pm\sqrt{1-\xi\theta^2} \quad \text{with} \quad \xi = \frac{\eta'^2}{\varepsilon^4}. \tag{3}$$

We note that 1DSfT cannot be solved uniquely, but we prove that it has a discrete amount of at least two solutions and this bound is given by the number of critical points. We explain how to detect these critical points and we prove some properties about their geometric interpretations and the sharing of critical solutions between solutions. For instance, at critical points, θ is recoverable uniquely and so is the depth φ_y. Then, between two consecutive critical points, the sign of θ' is constant.

These propositions allow us to apply the Picard-Lindelöf (PL) theorem in each interval bounded by two consecutive critical points. This theorem gives the existence and the uniqueness of solutions in first-order ODEs with initial conditions, given here by the critical points. Thanks to the PL theorem, we know the maximum number of solutions between two consecutive critical points, and, thanks to the number of critical points, we know how many times the PL theorem can be applied along the 2D curve. The number of solutions M can be shown to be $M \leq 2^{N+1}$, with N the number of critical points.

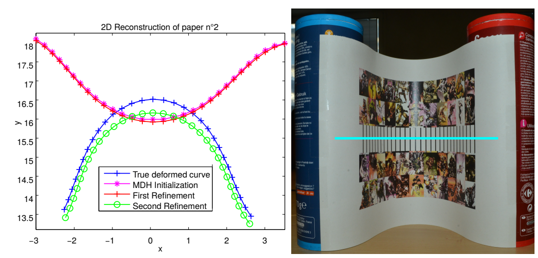

Figure 2: Real data experiments. Example of results with real data - Image used to obtained the 1D image.

The paper describes how we implement this theory to obtain isometric solutions to 1DSfT. We adapt and test existing 2DSfT methods on simulated and real data. We propose two convex initialization algorithms, a local analytical one based on infinitesimal planarity and a global one based on inextensibility, the Maximum Depth Heuristic (MDH) [4]. We refine the MDH solution with a nonlinear least-squares optimization. At this step, the isometric parameterization and the reprojection error are used to compute the cost function. Finally, we use the solution to find the critical points and compute the other solutions, as figure 2 shows.

Our conclusion is that there is no local exact solution and 1DSfT cannot be solved uniquely. The complexity of 1DSfT is revealed by the PDE analysis that provides thanks to the critical points and the PL theorem a bound on the solution space and a way to obtain solutions to 1DSfT.

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This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.