Reweighted Laplace Prior Based Hyperspectral Compressive Sensing for Unknown Sparsity

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Hyperspectral compressive sensing(HCS) [6] is newly proposed for hyperspectral image(HSI) compression, which compresses image during image acquisition. It can dramatically reduce the consumption of imaging resource compared with traditional compression methods such as DPCM, JPEG, etc. For HCS, how to recover the original HSI from a few measurements is a challenging problem. However, most of the existing norm-based or prior-based methods [2, 3] assume the sparse coefficients are independent, which neglect the structure information of sparse coefficients. Moreover, the adaptability of the regularization term(e.g. ℓ_0, ℓ_1 norm) to the unknown noise is unsolved.

To model the distribution of sparsity in HSI and make the method adaptive to the unknown noise, we propose a novel matrix-based reweighted Laplace prior by utilizing the characteristics of HSI. Then, a latent variable Bayes model [8] is employed to learn the hyperparameters of the reweighted Laplace prior from the measurements. This Bayes model unifies the signal recovery, prior learning and noise estimation into a variational framework. The learned sparsity prior can capture the underlying structure of the sparse signal and is adaptive to the unknown noise, which improves the reconstruction accuracy.

Specifically, based on the HCS model $G = AX + N$, the reconstruction is to recover the HSI X from measurements G given the random sampling matrix A. A linear basis matrix D is often introduced to transform X into a sparse signal Y as $X = DY$. Thus, we have the likelihood $p(G|Y,\Sigma_n) = \mathcal{MN}(ADY,\Sigma_n,I)$, where $\Sigma_n = diag(\lambda)$. To recover X, a hierarchical reweighted Laplace sparsity prior is proposed to model the sparsity of HSI. First, we represent the sparse signal Y with a matrix normal distribution as

$$p(Y|\gamma) = \frac{\exp\left\{-\frac{1}{2}\|Y\|^2_{\Sigma_y}\right\}}{(2\pi)^{n_b n_p/2}\left|\Sigma_y\right|^{n_p/2}}, \quad \Sigma_y = diag(\gamma). \tag{1}$$

Then, a Gamma distribution is imposed on the unknown γ as follows

$$p(\gamma|\kappa) = \prod_{i=1}^{n_b} \text{Gamma}(1, \frac{2}{\kappa_i}) = \prod_{i=1}^{n_b} \frac{\kappa_i}{2} \exp\left(-\frac{\kappa_i \gamma_i}{2}\right). \tag{2}$$

It can be proved that the proposed hierarchical prior equals to a reweighted Laplace prior as

$$p(y_i) \propto \int p(y_i|\gamma)\, p(\gamma|\kappa) d\gamma = \frac{\exp(-\|Ky_i\|_1)}{2^{n_b}|K|^{-1}}, K = diag\left([\sqrt{\kappa_1}, ..., \sqrt{\kappa_{n_b}}]^T\right). \tag{3}$$

To fit the specific distribution of HSI, a latent variable Bayes model is employed to learn the sparsity prior from measurements, which is polluted by unknown noise.

$$\max_{\lambda\geq 0,\gamma\geq 0,\kappa} p(\lambda,\gamma,\kappa|G) = \max_{\lambda\geq 0,\gamma\geq 0,\kappa} \int p(G|Y,\lambda)\, p(Y|\gamma)\, p(\gamma|\kappa)\, dY \tag{4}$$

This optimization equals to

$$\min_{\lambda\geq 0,\gamma\geq 0,\kappa} \mathbf{tr}\left(n_{n_p}^{-1} G^T \Sigma_{by}^{-1} G\right) + \log\left|\Sigma_{by}\right| + \sum_{i=1}^{n_b} \frac{\kappa_i \gamma_i - 2\log\kappa_i}{n_p} \tag{5}$$

where $\Sigma_{by} = \Sigma_n + AD\Sigma_y D^T A^T$. According to Ref. [8], we further unify signal recovery, sparsity learning and noise estimation into one framework

$$\min_{Y,\lambda\geq 0,\gamma\geq 0,\kappa} \left\|ADY - \frac{G}{\sqrt{n_p}}\right\|^2_{\Sigma_n} + \|Y\|^2_{\Sigma_y} + \log\left|\Sigma_{by}\right| + \frac{1}{n_p}\sum_{i=1}^{n_b}(\kappa_i\gamma_i - 2\log\kappa_i) \tag{6}$$

This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.

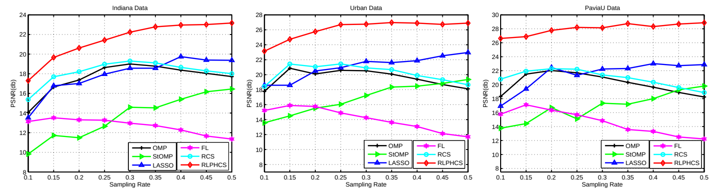

Figure 1: PSNR curves on three datasets under SNR=$10db$.

	$\rho = 0.10$	$\rho = 0.20$	$\rho = 0.30$	$\rho = 0.40$	$\rho = 0.50$
OMP [7]	0.2632	0.3666	0.3453	0.2855	0.2357
StOMP [4]	0.0169	0.1720	0.2741	0.3469	0.4207
LASSO [5]	0.3526	0.5312	0.5512	0.5459	0.5289
FL [1]	0.1519	0.1780	0.1343	0.1019	0.0768
RCS [3]	0.3638	0.4132	0.3697	0.3081	0.2525
RLPHCS	**0.7142**	**0.7819**	**0.8177**	**0.8217**	**0.8294**

Table 1: Average SSIM on three datasets under SNR = $10db$.

The learned sparsity prior can capture the underlying structure of the sparse signal and adaptive to the unknown noise in CS procedure. Moreover, the proposed reweighted Laplace prior can reduce the undemocratic penalization effect of traditional Laplace prior(See Section 2.4).

We evaluate the performance of the proposed RLPHCS on three real hyperspectral datasets, including INDIANA, URBAN and PAVIAU. Five state-of-the-art compressive sensing methods are employed as the comparison methods. The comparison results of PSNR and average SSIM scores on three datasets under SNR = $10db$ are shown in Figure. **??** and Table **??**. The experimental results indicate the superiority of the proposed RLPHCS on the reconstruction accuracy of HSI.

In this paper, we propose a novel reweighted Laplace prior based HCS method. The learned prior depicts the underlying structured sparsity of HSI very well and is adaptive to unknown noise. This helps the algorithm reconstruct the HSI precisely.

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