A Solution for Multi-Alignment by Transformation Synchronisation

Florian Bernard[1,2,3], Johan Thunberg[2], Peter Gemmar[3], Frank Hertel[1], Andreas Husch[1,2,3], Jorge Goncalves[2]
[1]Centre Hospitalier de Luxembourg, Luxembourg. [2]Luxembourg Centre for Systems Biomedicine, University of Luxembourg, Luxembourg. [3]Trier University of Applied Sciences, Germany.

The alignment of a set of objects by means of transformations plays an important role in computer vision. The case of aligning only two objects is known as *Absolute Orientation Problem (AOP)* [5] or *Procrustes Analysis* and can be solved globally. However, when multiple objects are considered, which is known as *Generalised Procrustes Analysis (GPA)*, usually iterative methods are used. In practice the iterative methods perform well if the relative transformations between any pair of objects are free of noise. However, if only noisy relative transformations are available (e.g. due to missing data or wrong correspondences) the iterative methods may fail.

In this paper a method for denoising the set of pairwise relative transformations is presented. Previously, Singer et al. [2, 4, 6] have presented a solution for denoising the set of orthogonal transformation matrices. However, in this work we consider the more general case of invertible linear transformations. For that we make use of the fact that the noise-free transformations must fulfil the transitive consistency condition, i.e. transforming object A to B and B to C must be identical to directly transforming A to C.

The invertible pairwise transformation matrices $\mathbf{T}_{ij} \in \mathbb{R}^{d \times d}$ between all k objects can be arranged into a single large matrix $\mathbf{W}$, i.e.

$$\mathbf{W} = [\mathbf{T}_{ij}] \tag{1}$$

$$= \begin{bmatrix} \mathbf{T}_{11} & \cdots & \mathbf{T}_{1k} \\ \vdots & \ddots & \vdots \\ \mathbf{T}_{k1} & \cdots & \mathbf{T}_{kk} \end{bmatrix} \tag{2}$$

$$= \begin{bmatrix} \mathbf{T}_{1\star}\mathbf{T}_{1\star}^{-1} & \cdots & \mathbf{T}_{1\star}\mathbf{T}_{k\star}^{-1} \\ \vdots & \ddots & \vdots \\ \mathbf{T}_{k\star}\mathbf{T}_{1\star}^{-1} & \cdots & \mathbf{T}_{k\star}\mathbf{T}_{k\star}^{-1} \end{bmatrix}. \tag{3}$$

In the case of transitively consistent transformations the matrix $\mathbf{W}$ can be factorised as $\mathbf{W} = \mathbf{U}_1\mathbf{U}_2$, with

$$\mathbf{U}_1 = \begin{bmatrix} \mathbf{T}_{1\star} \\ \mathbf{T}_{2\star} \\ \vdots \\ \mathbf{T}_{k\star} \end{bmatrix} \quad \text{and} \quad \mathbf{U}_2 = \left[\mathbf{T}_{1\star}^{-1}, \mathbf{T}_{2\star}^{-1}, \ldots, \mathbf{T}_{k\star}^{-1}\right],$$

where $\star$ denotes *some* reference coordinate frame from which there are $\mathbf{T}_{i\star}$ transformations such that $\mathbf{T}_{ij} = \mathbf{T}_{i\star}\mathbf{T}_{\star j}$ for all i, j. We demonstrate that this factorisation can be retrieved from the d-dimensional null space of the matrix $\mathbf{Z} = \mathbf{W} - k\mathbf{I}$.

For the case of a noisy observation of $\mathbf{W}$, the least-squares approximation of the d-dimensional null space is considered instead, resulting in the set of *synchronised* transformations that are transitively consistent.

Our simulations demonstrate that for noisy pairwise transformations the error to ground truth transformations can be reduced by applying the proposed synchronisation method. Furthermore, we apply the method to solve the Generalised Procrustes Problem (GPP) for the case of missing data as well as for the case of wrong correspondence assignments. A subset of the simulations for solving the GPP in the case of wrong correspondence assignments using the 2D fish shapes from the Chui-Rangarajan data set [3] are shown in Fig. 1. Various methods for solving the GPP are evaluated. In the reference-based method one shape is randomly selected as reference and all other shapes are aligned with the reference. For the iterative mean shape-based method the initial reference shape is selected randomly and then the mean shape is iteratively updated. In the synchronisation-based solution of the GPP all k^2 pairwise AOPs are solved first, followed by the synchronisation of the resulting transformations using the proposed method in order to aggregate all information contained in the set of pairwise transformations. Additionally, the stratified GPA method presented in [1] is evaluated for solving the GPP. For the missing data experiments our method outperforms the other approaches since it is the only one that is able to make use of the information contained in all pairwise transformations.

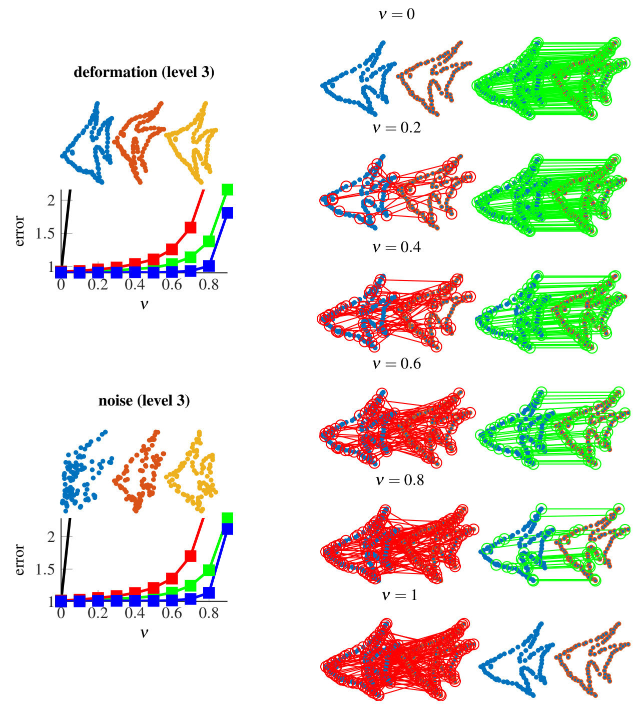

Figure 1: Left: Example of the average shape error for the reference-based (green), iterative mean shape-based (black), synchronisation-based (blue) and stratified (red) method for solving the GPP for a fraction v of wrong correspondences at a particular level of deformation (top) or noise (bottom). Shown is the average shape error for 500 runs of disturbing correspondence assignments. Right: Examples of possible assignments between a pair of shapes for different values of wrong correspondences v. In order to keep the visualisation as coherent as possible, the wrong correspondences (red lines) and the correct correspondences (green lines) are shown separately.

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This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.