Robust Multiple Homography Estimation: An Ill-Solved Problem

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Images in two views of world points lying on a planar surface are related by a homography matrix. Planar surfaces are ubiquitous in urban environments, and this makes estimating multiple homography matrices from image measurements between two views an important step in many applications such as augmenting reality, stitching and warping images, calibrating cameras, finding a metric reconstruction, and detecting non-rigid motion. Because of the diverse utility of homography matrices, researchers often exploit the task of estimating multiple homographies to demonstrate the merits of robust multi-structure estimation methods. In fact, some robust multi-structure estimation methods, such as *multiRANSAC* [5], were specifically designed to address the multi-homography estimation problem. However, an inadvertent oversight has crept into multi-structure estimation methods, one that persists in all state-of-the-art methods that we are familiar with, namely the failure to recognise that a set of homographies that each of these schemes produces is actually *not* a genuine set of homographies between two views of the same scene. A collection of homography matrices forms a valid set only if the matrices satisfy consistency constraints implied by the rigidity of the motion and the scene. If the constraints are not deliberately enforced, they are not satisfied in typical scenarios. Hence, one of the fundamental problems in estimating multiple homography matrices is to find a way to enforce the consistency constraints—a task reminiscent of that of enforcing the rank-two constraint in the case of the fundamental matrix estimation.

The consequence of homographies being incompatible can clearly be appreciated by examining the extent to which a fundamental matrix constructed from an incompatible set of homographies can represent the epipolar geometry of the scene. As is well known, given a homography $\mathbf{H}$ between the first and second views induced by a plane in the scene, the underlying fundamental matrix $\mathbf{F}$ can be written as

$$\mathbf{F} = \mathbf{H}^{-\top}[\mathbf{e}]_{\times}, \tag{1}$$

where $\mathbf{e}$ represents the epipole in the first image. If $\{\mathbf{H}_i\}_{i=1}^{I}$ is a set of homographies, then, as it turns out, the epipole in the first image is a fixed point of any homology with matrix of the form $\mathbf{H}_j^{-1}\mathbf{H}_i$ and can be retrieved as the eigenvector corresponding to a non-repeated eigenvalue of any of the $\mathbf{H}_j^{-1}\mathbf{H}_i$'s. If the homography matrices $\mathbf{H}_i$ are incompatible, the various homology matrices $\mathbf{H}_j^{-1}\mathbf{H}_i$ will also be incompatible and will lead to different estimates of the epipole in the first image. This in turn will lead to different estimates of the fundamental matrix, as exemplified in Figure 1. Moreover, if a fundamental matrix is constructed by selecting a particular homography matrix $\mathbf{H}_{i_0}$ in (1), then even though the resulting fundamental matrix may be compatible with $\mathbf{H}_{i_0}$, it will not be compatible with the remaining homographies $\mathbf{H}_i$, $i \neq i_0$. Hence, given multiple incompatible homographies, it is possible to construct multiple incompatible fundamental matrices. This subtlety is overlooked in all robust multi-structure estimation methods that we are familiar with.

Explicit formulae for all constraints that must be satisfied in order for a collection of homographies to form a valid set have so far eluded the vision community. It was only as recently as 2011 that a decisive answer pertaining to even just the *number* of constraints was given [1, 2]. There are several reasons why knowledge of explicit formulae for homography constraints would be advantageous: (1) with explicit constraints it would be possible to devise new homography estimation methods that enforce full compatibility without recourse to latent variables; (2) one could investigate new global optimisation methods for multi-homography estimation, analogous to what has recently been achieved for fundamental matrix estimation [3, 4]; and (3) one could establish a new generation of robust multi-structure fitting methods that, unlike existing methods, yield estimates with consistent epipolar geometry.

This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.

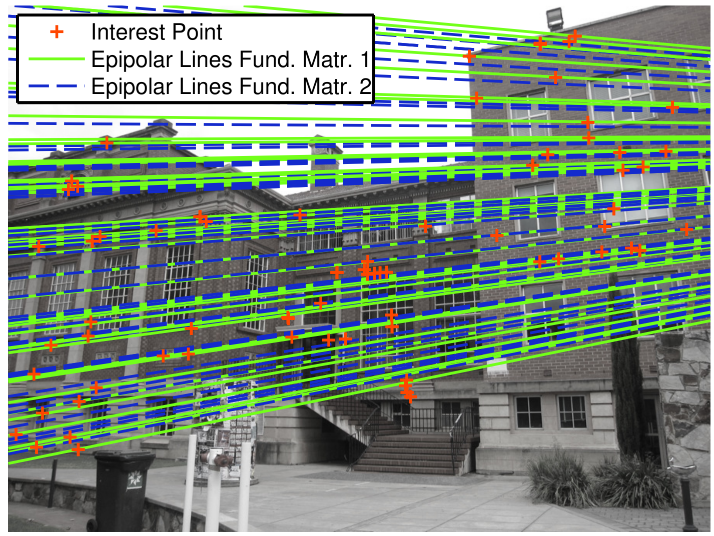

Figure 1: Comparison of epipolar lines associated with two fundamental matrices computed from two separately estimated homographies. The epipolar lines associated with the two fundamental matrices do not overlap, and thereby demonstrate that the two homographies do not share the same epipolar geometry and hence are incompatible.

In this paper we take a step toward achieving the goal of determining all homography constraints. We derive two new sets of consistency constraints and use them to measure, for the first time, the extent to which separately estimated homographies are mutually incompatible. Our findings indicate that none of the state-of-the-art multi-structure estimation methods adequately address the problem of multiple homography estimation, as none of them enforce consistency constraints. Consequently, robust multiple homography estimation continues to be an ill-solved problem.

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