Bayesian Inference for Neighborhood Filters with Application in Denoising

Chao-Tsung Huang
National Tsing Hua University, Taiwan

Range-weighted neighborhood filters, such as Yaroslavsky filter, bilateral filter [6] and non-local means [2], are useful and popular for their edge-preserving property and simplicity, but they are originally proposed as intuitive tools. For understanding their properties, previous works needed to connect them to other classical methods, e.g. mean shift [1], anisotropic diffusion [3] and robust estimation [4]. However, these connections were unable to provide further information to infer the range variance σ_r^2 directly from the observed data. On the other hand, statistical techniques have been adopted to estimate the parameters without reasoning the properties. Although SURE-based methods [5, 7] can achieve high prediction accuracy, the complexity is quite high because many filtering iterations are required.
In this paper, we build a unified empirical Bayesian framework to both infer the neighborhood filters and estimate their range variance. We first introduce a novel neighborhood noise model (NNM):

$$\mathbf{y}_i = \mathbf{z}_l + \frac{\mathbf{n}_{l,i}}{\sqrt{w_{l,i}}}, \tag{1}$$

where $\mathbf{z}_l$ is the latent signal at position l, $\mathbf{y}_i$ are the observed signals in its neighborhood Λ_l, and $\mathbf{n}_{l,i}$ are additive white Gaussian noises (AWGN) of covariance matrix $\sigma^2\mathbf{I_k}$. Especially, $w_{l,i}$ can model localized intensity edges using soft decisions. Namely, a smaller $w_{l,i}$ means that $\mathbf{y}_i$ is distributed more widely and thus there is more likely an edge between positions l and i. For inferring the Gaussian range-weighted kernel, $w_{l,i\neq l}$ are defined as white hidden variables of a prior distribution:

$$f_w(w;\varepsilon,\alpha) = \frac{1}{N(\varepsilon,\alpha)} w^{-k/2} w^{-\alpha w} e^{\alpha w},\ w \in [\varepsilon, 1], \tag{2}$$

where $N(\varepsilon,\alpha) = \int_\varepsilon^1 w^{-k/2} w^{-\alpha w} e^{\alpha w}\, dw$ for normalization.
Then by minimizing the energy function L_l, we have

$$\frac{\partial L_l}{\partial w_{l,i}} = 0 \quad \Rightarrow w_{l,i} = e^{-\frac{g_{l,i}^2}{2\sigma_r^2}},\ \sigma_r^2 = \alpha\sigma^2,\ (i \neq l) \tag{3}$$

$$\frac{\partial L_l}{\partial \mathbf{z}_l} = 0 \quad \Rightarrow \mathbf{z}_l = \frac{\sum_{i\in\Lambda_l} w_{l,i}\cdot \mathbf{y}_i}{\sum_{i\in\Lambda_l} w_{l,i}}, \tag{4}$$

where $g_{l,i}^2 \triangleq \| \mathbf{z}_l - \mathbf{y}_i \|_2^2$. Thus the Yaroslavsky filter is equivalent to the first-iteration estimation for solving this fixed-point problem with an initial condition $\mathbf{z}_l^{(0)} = \mathbf{y}_l$. The above formulations can be further interpreted into two sequential steps: 1) Maximum a posteriori (MAP) estimation for $w_{l,i}$ in (3); 2) Maximum likelihood (ML) estimation in (4). By modifying the likelihood function in the second step for considering proximity and patch similarity, the bilateral and a modified non-local means filters can be derived respectively.
We then present an iterative EM+ algorithm to perform the model fitting for estimating the range variance σ_r^2 from noisy images. Let $s_{l,i} \triangleq \| \mathbf{y_l} - \mathbf{y_i} \|_2$. It is independent of $\mathbf{z_l}$ and thus observable. It follows the chi scale mixtures (CSM) formulation:

$$s_{l,i} = \sigma\sqrt{\frac{w_{l,i}+1}{w_{l,i}}}\, u_{l,i},\ u_{l,i} \sim \chi_k, \tag{5}$$

where $u_{l,i}$ is a chi distribution with k degrees of freedom. Then for any given observed data set $\mathcal{S}$, we can estimate the corresponding pdf $f_s(s)$ by iteratively updating the model parameters $(\sigma,\alpha,\varepsilon)$ based on the empirical distribution $P(s\in\mathcal{S})$. Each iteration of the EM+ algorithm consists of three steps: EM update for (σ,α), KLD update for ε, and Quasi-Newton (QN) update for acceleration.

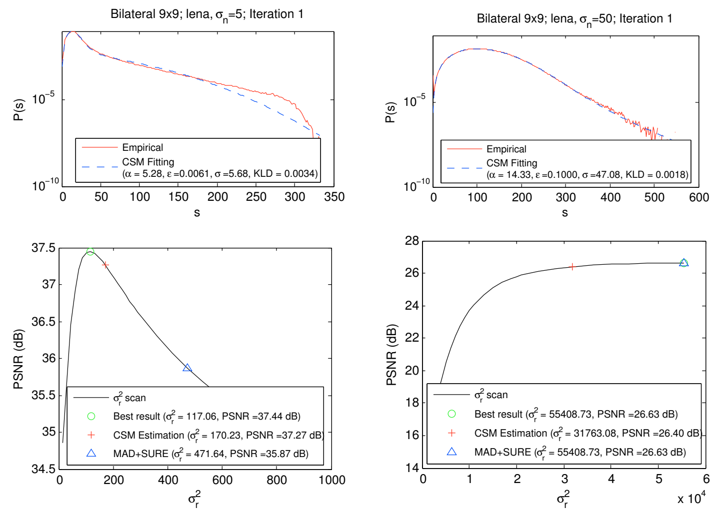

Figure 1: Bilateral filter test for *Lena* with noise intensity $\sigma_n = 5$ and 50. The top row shows the model fitting of empirical distributions, and the bottom presents the estimation accuracy.

We apply the proposed framework to the color-image denoising problem for validating the effectiveness. Experimental results show that the proposed model fits noisy images well and estimates σ_r^2 as accurate as SURE does. Fig. 1 shows a typical example, and all the results can be browsed online[1]. We also show that the model fitting even works for filtered images for which the noise is no longer Gaussian. This useful property enables a recursive filtering scheme which can further improve image quality. Therefore, the advantage of the proposed framework over SURE is both computation-wise and quality-wise because SURE is slow and fails for the filtered images.
The proposed framework can be used to build efficient filters for different constraints automatically, instead of heuristic tuning. It can also be expected that it will be applied to other range-weighted algorithms by formulating the corresponding likelihood functions. Therefore, we believe that it will help many computer vision problems be solved in an empirical Bayesian way, instead of an intuitive way.

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This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.

[1] Project website: `http://www.ee.nthu.edu.tw/chaotsung/nnm`