A Linear Least-Squares Solution to Elastic Shape-from-Template

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Method	Deformation			Time	Acc	Noise	Init/params
	Isometric	Conformal	Elastic				
analytic-GbM-[2]	+	+	−	+	−	−	+
non-linear-GbM-[3]	+	+	−	−	+	+	−
non-linear-MbM-[4]	+	+	+	−	+	+	−
sequential-MbM-[1]	+	+	+	+	−	+	−
linear-MbM-Proposed	+	+	+	+	+	+	+

Table 1: Strong (+) and weak (-) points of state-of-the-art physics-based SfT methods. GbM stands for geometric method and MbM for mechanical methods. **Time** is the computational cost. **Acc** is the accuracy of the method to recover the deformed shape. **Noise** is the sensitivity of the method to image noise. **Init/params** tells if the method requires an initial shape or hyper-parameters (-) or not (+). Isometric SfT methods are limited to isometric deformations and are not mentioned.

SfT (Shape-from-Template) is to reconstruct a 3D deformed surface from its image, a 3D shape template and some priors among the physics or the statistics which rule the deformation. In physics-based priors, the geometric and mechanical priors are the most used. This paper uses the latter one. While geometric priors tend to use invariant measures between deformations, mechanical priors represent the physics that rules the deformation. Such methods have the potential for real applications since they address a wider class of deforming objects such as organs, tissues and other elastic objects.

Current geometric/mechanical SfT methods suffer from caveats that are summarized in table 1. First, the types of deformations: geometric approaches [2, 3] may not behave well when the prior is not fulfilled by the deformation, while mechanical methods [1, 4] cover a larger range of deformations (such as linear elasticity). Second, the tradeoff between accuracy and speed of estimation. Analytical solutions [2] are fast but with lower accuracy. Non-linear optimization methods [3, 4] are more accurate but at a higher computation cost. Moreover, their accuracy depends on the initialization and on some hyper-parameters. Methods based on Kalman filtering [1] also require an initialization step besides the fact that errors may accumulate over the considered time frame. In summary, physics-based approaches lack a method which is *(i)* able to exploit the mechanical constraints to cover a large deformation range, *(ii)* robust to noise, and *(iii)* both accurate and fast. Developing such a method is the main goal of this work. The answer we propose is to use mechanical constraints into a linear least-squares estimation framework.

We use finite elements (FEM) to represent the surface and the deformation. This is particularly adapted to SfT and fits the problem we want to solve. Indeed, only the finite discrete set of point correspondences has a natural boundary condition through the *RBC* (Reprojection Boundary Constraint). Any other point of the surface is free from boundary conditions and is only subject to the physical prior. Thus, we use the finite set of correspondences as nodes of the elements (triangles). Each element is subject to the mechanical laws that rule the deformation. We use a weak formulation of these laws to derive a linear relation (via the stiffness matrix) between the displacement of the nodes and the external deforming forces. The assembly of all elementary matrices reconstruct the global stiffness matrix that links the global external deforming force to the global deformation $\delta\mathbf{X}$. We further constrain the problem with a set of *SBC* (Solid Boundary Constraints) to rigidly position the template in the deformed image. Minimizing this force subject to the *RBC* and *SBC* is a linear least-squares problem that can be solved very efficiently.

Let n be the number of point correspondences. We propose to formalize

This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.

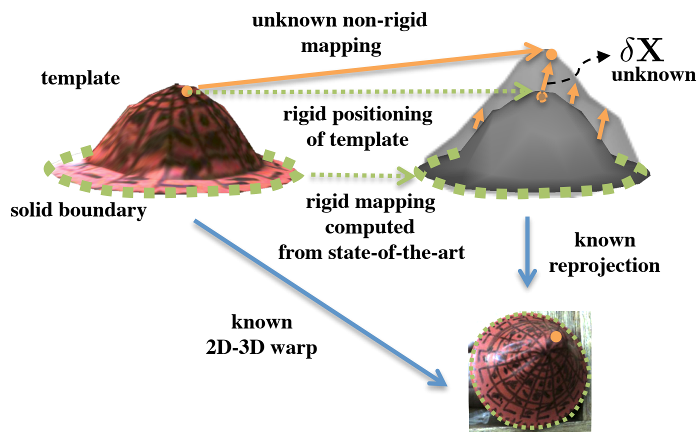

Figure 1: New formalization of the physics-based SfT. The displacement field at the solid boundary point is the zero 3-vector.

SfT as follows: *find the 3D displacement field* $(\mathbf{X}, \delta\mathbf{X}) \in \mathbb{R}^{3n} \times \mathbb{R}^{3n}$*, where:* (i) $\delta\mathbf{X}$ *minimizes the norm of the applied forces for the deformation,* (ii) $\mathbf{X} + \delta\mathbf{X}$ *satisfies the RBC, and* (iii) $\mathbf{X}$ *is a rigid positioning of the template shape in the deformed frame thanks to the SBC.* See figure 1 for an illustration. Formally speaking, finding $(\mathbf{X}, \delta\mathbf{X})$ involves two main steps:

1. Find $\mathbf{X}$ from $\mathbf{X}_0$ with a PnP method applied to the solid boundary points of *SBC*. $\mathbf{X}_0$ is the template pose in the world coordinate frame.

2. Find $\delta\mathbf{X}$ such that:

$$\min_{\delta\mathbf{X}\in\mathbb{R}^{3n}} \frac{1}{2} \parallel \mathbf{K}\,\delta\mathbf{X} \parallel^2 \text{ s.t.} \begin{cases} \mathbf{P}\,\delta\mathbf{X} = \mathbf{b} & RBC \\ \mathbf{S}\,\delta\mathbf{X} = \mathbf{0} & SBC \end{cases} \tag{1}$$

Matrix $\mathbf{K}$ is the stiffness matrix of size $3n \times 3n$. Note that $\mathbf{f} = \mathbf{K}\,\delta\mathbf{X}$ is thus the vector of external forces to be minimized. $\mathbf{P}$ is an n-block diagonal matrix of dimension 2×3 per block. $\mathbf{P}$ and $\mathbf{b}$ enforce the fact that the n points must lie on the sightlines that pass through the n corresponding points in the deformed image. $\mathbf{S}$ is an $m \times 3n$ sparse matrix. It adds m depth displacement constraints which together with the corresponding $2m$ equations of *RBC* set the m solid boundary points. In the paper, we describe how to compute $\mathbf{K}$ and we show that if it is full rank, then there exists a unique solution to this problem.

In the paper, we test the proposed method on a various deformations using synthetic and real datasets. We compared the obtained results to the methods referenced in table 1. Our conclusion, is that the linear formulation together with mechanical priors covers a large panel of deformations and brings efficiency in both computation cost and accuracy.

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