A Statistical Model of Riemannian Metric Variation for Deformable Shape Analysis

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The analysis of deformable 3D shape is often cast in terms of the shape's intrinsic geometry due to its invariance to a wide range of non-rigid deformations. However, object's plasticity in non-rigid transformation often result in transformations that are not completely isometric in the surface's geometry and whose mode of deviation from isometry is an identifiable characteristic of the shape and its deformation modes.

In this paper, we propose a new supervised technique to learn a statistical model build on the Riemannian metric variations on deformable shapes based on the spectral decomposition of the Laplace-Beltrami operator. To this end, we define a statistical framework that models a shape as two independent models for the eigenvectors and for the eigenvalues. The eigenvector matrices of a set of discrete representations (*i.e.* meshes representing the shape in different poses) are assumed to be points on the manifold of special orthogonal matrices $\mathcal{SO}(n)$. Here we assume the model to follow a Γ-distribution over the manifold geodesic distances from a manifold centroid Φ_0

$$d_g(\phi,\phi_0) \approx 2n - Tr\left(\phi^T\phi_0\right) + O(\Theta_i^4) \quad (1)$$

where Θ_i are the angles of the residual rotation $\phi^T\phi_0$.

On the other hand, we assume that the eigenvalues are log-normally distributed for the same stability considerations presented by Aubry *et al.* [1]. The shape centroid is computed as follows:

$$\underset{\phi_0,\mathcal{R}_i\in\mathcal{SO}(p)}{\text{argmax}} \quad \text{Tr}\left(\sum_i^N \mathcal{R}_i\phi_i^T\phi_0\right) \quad (2)$$

where the rotation matrix $\mathcal{R}_i$ is introduced in order to align the eigenvectors of the Laplacian of a mesh i, since its embedding is defined up to an isometry. This is solved by separately optimizing for ϕ_0 and $\mathcal{R}_i$ in an iterative process

$$\Phi_0 = \underset{\Phi_0\in\mathcal{O}(n)}{\text{argmax}} \quad \text{Tr}\left(\left(\sum_i^N \mathcal{R}_i\Phi_i^T\right)\Phi_0\right) \quad (3)$$

$$\mathcal{R}_i = \underset{\mathcal{R}_i\in\mathcal{SO}(n)}{\text{argmax}} \quad \text{Tr}\left(\left(\sum_i^N \Phi_i^T\Phi_0\right)\mathcal{R}_i\right) \quad (4)$$

where both optimizations can be solved exactly through Singular Value Decomposition.

We tested our method on several standard shape retrieval datasets, showing that the proposed approach is competitive with the current state-of-the-art for non-rigid 3D shape retrieval.

Table 1: Comparison of different retrieval methods, in terms of average precision on the SHREC'10 [2] datasets, broken down according to different transformations.

Transformation	VQ	Sup. DL	RMVM
Isometry	98.8	**99.4**	**99.4**
Topology	100	100	100
Isometry+Topology	93.3	95.6	**99.5**
Partiality	94.7	**95.1**	90.0
Triangulation	95.4	95.5	**96.5**

This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.

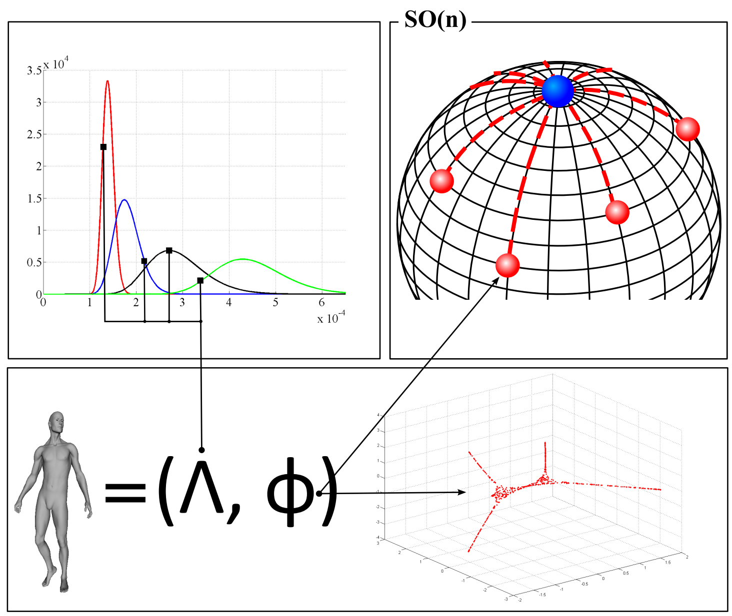

Figure 1: Example of the learning process of the proposed method, from the computation of the spectral decomposition of the Laplacian associated to a mesh to the definition of the two separate independent models employed in the inference phase.

Table 2: Comparison of different retrieval methods in terms of mean average precision on the SHREC'14 [3] Humans datasets.

Method	Synthetic	Scanned
ISPM	90.2	25.8
DBN	84.2	30.4
R-BiHDM	64.2	64.0
HAPT	81.7	63.7
ShapeGoogle (VQ)	81.3	51.4
Unsupervised DL	84.2	52.3
Supervised DL	95.4	79.1
RMVM	**96.3**	**79.5**

References

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