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MRF Optimization by Graph Approximation

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The right choice of proposals is of crucial importance for the success of graph-cuts based algorithms. In this paper, we propose a generic and application-independent approach to generate good proposals for non-submodular energy functions. With these proposals, we present a graph cuts-based move-making algorithm called GA-fusion (fusion with graph approximation-based proposals). This method is simple but powerful. It is applicable to any type of energy functions. The basic idea of the overall algorithm is depicted in Figure 2, which illustrates a single iteration of the proposed algorithm.

Given a graph $G = (\mathcal{V}, \mathcal{E})$, the energy function of the pairwise MRF is given by

$$E(\mathbf{x}) = \sum_{p \in \mathcal{V}} \theta_p(x_p) + \lambda \sum_{(p,q) \in \mathcal{E}} \theta_{pq}(x_p, x_q), \tag{1}$$

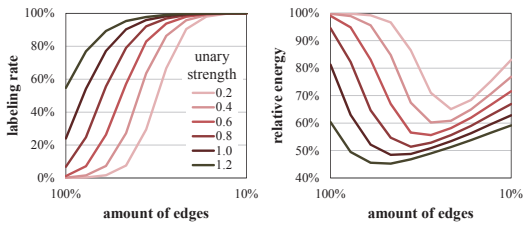
Graph cuts-based move-making algorithms [1, 3] have achieved great success in minimization of the energy function (1). Although they have been successful for many different applications, they still end up with unsatisfactory solutions when it comes to extremely difficult problems. In those kind of problems, many graph cuts-based algorithms cannot label sufficient number of nodes due to the strong non-submodularity.

Although there has been a demand for a generic method of proposal generation, little research has been done on the mechanism of "good" proposal generation. Instead, most research on proposal generation is often limited to application-specific approaches [2, 4]. The three properties for "good" proposals can be summarized as follows:

- **Quality** Good proposals are close to minimum such that proposals can guide the solution to minimum by fusion moves. In other words, good proposals have low energy.
- **Diversity** For the success of the fusion approach, diversity among different proposals is required.
- **Labeling rate** Good proposals result in high labeling rate when they are fused with the current solution. In other words, good proposals produce easy-to-solve sub-problems.

We approximate the original function $E(\mathbf{x})$ to achieve aforementioned properties for good proposals. Two conditions are required for an approximated function $E'(\mathbf{x})$. First, the approximated function should be easy to solve although the original one $E(\mathbf{x})$ is difficult. In other words, more nodes are labeled when we apply simple α-expansion algorithm. Second, the approximated function should be similar to the original one. In other words, solution $\mathbf{x}'$ of the approximated function should have low energy in terms of the original function.

Our algorithm approximates original target function simply by dropping random subset of edges. The approximated function is



(a) (b)

Figure 1: (a) Labeling rates and (b) relative energies are depicted as the graph is approximated with a random subset of edges. Relative energies are calculated with the original functions.

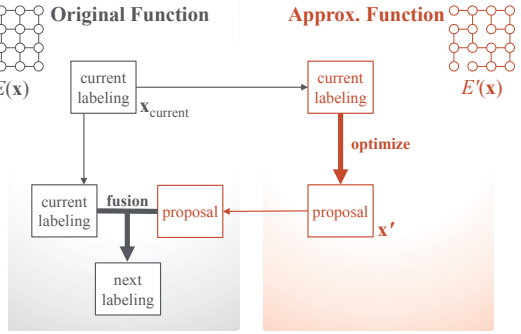


Figure 2: The basic idea of the overall algorithm. The original function is approximated via graph approximation. The approximated function is optimized, and the solution is used as a proposal for the original problem.

$$E'(\mathbf{x}) = \sum_{p \in \mathcal{V}} \theta_p(x_p) + \lambda \sum_{(p,q) \in \mathcal{E}'} \theta_{pq}(x_p, x_q). \tag{2}$$

where a subset of pairwise terms $\theta_{pq}|_{(p,q) \in \mathcal{E} \setminus \mathcal{E}'}$ is dropped from the energy formulation (1).

Figure 1 shows the proposed approximation makes the problem easy to solve, as well as similar to the original function. We first built extremely difficult MRF models by high non-submodularity and coupling strength. And then label them using QPBO while approximating the original function. More nodes are labeled when the approximation uses a smaller subset $\mathcal{E}'$. Interestingly, energies at appropriate amount of edges are even lower than using original functions. The balance between the difficulty and the similarity to the original function can be adjusted by changing the size of the subset $\mathcal{E}'$.

The final result from the deconvolution problem is shown in Fig. 3. Proposed method achieves lowest energy while other graph-cuts based ones are stuck at local minima due to the inappropriate choice of the proposals. Our through experiments demonstrate that the proposed approach significantly outperforms existing algorithms for non-metric functions and competitive with other state-of-the-art for metric functions.

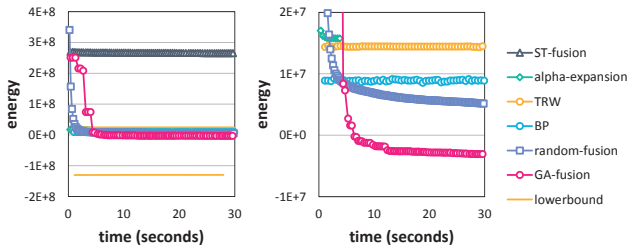


Figure 3: Energy decrease of each method for the deconvolution of the Santa image. Two plots shows the same curves from a single experiment, with different scales on the y-axis.

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This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.