Transformation of Markov Random Fields for Marginal Distribution Estimation

Masaki Saito[1], Takayuki Okatani[1]
[1]Tohoku University, Japan.

Markov Random Fields (MRFs) have been used for a wide range of problems in computer vision. In this study, we propose a general method for efficiently solving their marginal inference problem by transforming the original MRF model into a smaller, simpler one. While many existing methods [1, 2, 4, 5, 6] that transform MRFs only focuses on the MAP estimation problem and empirically transform the energy function, our method systematically derives transformed MRFs suited for the marginal inference problem.

The core of our approach is as follow. Let the probability distribution of a MRF be $p_0(x) \propto \exp(-E_0(x))$, where $E_0(x)$ is an energy function of the MRF. We consider the case where the marginal inference problem with $p_0(x)$ is intractable or computationally costly. We propose to introduce a new variable z_1 and construct a simpler MRF model $p_1(z) \propto \exp(-E_1(z_1))$ where the energy is given by

$$E_1(z_1) = \sum_x q_{0,1}(x|z_1)\left\{E_0(x) + \ln q_{0,1}(x|z_1)\right\}. \tag{1}$$

Here $q_{0,1}(x|z_1)$ is a conditional distribution that we choose depending on applications. The procedure after choosing $q_{0,1}(x|z_1)$ is as follows.

1. Compute the energy function $E_1(z_1)$ from $E_0(x_0)$ and $q_{0,1}(x|z_1)$.
2. Compute the marginal distributions for the transformed MRF (having $E_1(z_1)$ as the energy) by using a selected algorithm (e.g., Mean Field Approximation, Belief Propagation, etc.).
3. Compute approximate marginal distributions of $p_0(x)$ from those of $p_1(z_1)$ obtained above.

An obvious issue is how to choose $q_{0,1}(x|z_1)$. The energy function $E_1(x)$ of the transformed MRF is determined by the variable space of z_1 and $q_{0,1}(x|z_1)$. Our method causes at least three practical applications (Fig.1), which are i) the discretization of variable space, ii) the grouping of discrete labels, and the iii) coarse graining of MRFs. The discretization of variable space transforms a MRF model which has a continuous variable and is impossible to derive marginal distributions into a simpler MRF having a discrete variable. The grouping of discrete labels groups multiple discrete labels into one label. The coarse graining of MRFs transforms graphs into smaller ones in such a way that a number of connected sites are grouped into a single site. The specific forms o $q_{0,1}(z|x)$ and $E_1(z_1)$ is given in our main paper. In the paper, we also show how some of these MRF transformations are combined in a coarse-to-fine manner, and how our MRF transformation approach is also applied to Markov chain Monte Carlo methods.

Through several experiments, we confirmed the effectiveness of our proposed method. For a grid pairwise CRF model we considered a problem that finds the best parameter representing the interaction between two sites. We used the MSRC-21 dataset [3] for the experiments, and applied the two of the above methods for downsizing CRF. The first is grouping discrete labels, where we reduced the number of labels to K for each pixel. The second is coarse graining of the MRF, where we downsized the original grid MRF by grouping the pixels in $b \times b$ square blocks into a single superpixel.

Table 1 shows quantitative results. The "disparity" column shows the mean differences of the parameter between the full MRF and its downsized versions. The "accuracy" column shows the percentage of correctly labeled pixels. This indicates that both two transformations succeeded in reducing the learning time while maintaining the accuracy.

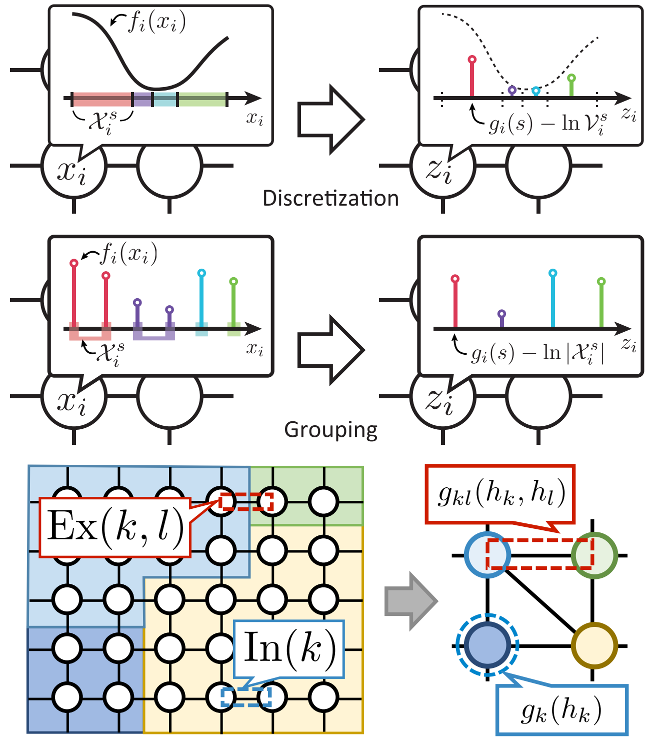

Figure 1: Three practical applications of our method. Upper row: discretization of continuous variable space. Middle row: grouping of discrete labels. Bottom row: coarse graining of MRFs.

Table 1: Quantitative results on the MSRC-21 dataset.

	time [h]	speedup	disparity	accuracy
full MRF	9.5	-	0.0	81.6
2 labels	0.89	10.6×	0.01235	77.8
3 labels	0.95	10.0×	0.00526	80.7
4 labels	1.0	9.2×	0.00496	81.3
5 labels	1.1	9.0×	0.00473	81.3
4×4 grid	0.66	14.4×	0.215	81.5
3×3 grid	1.1	8.5×	0.236	81.6
2×2 grid	2.5	3.9×	0.237	81.7

[1] Pedro F Felzenszwalb and Daniel P Huttenlocher. Efficient Belief Propagation for Early Vision. *IJCV*, 70(1):41–54, 2006. ISSN 0920-5691.

[2] Cheng Lei and Yee-Hong Yang. Optical Flow Estimation on Coarse-to-Fine Region-Trees using Discrete Optimization. In *ICCV*, 2009.

[3] Jamie Shotton, John Winn, Carsten Rother, and Antonio Criminisi. TextonBoost for Image Understanding: Multi-Class Object Recognition and Segmentation by Jointly Modeling Texture, Layout, and Context. *IJCV*, 81(1), 2009.

[4] Olga Veksler. Reducing Search Space for Stereo Correspondence with Graph Cuts. In *BMVC*, 2006.

[5] Liang Wang, Hailin Jin, and Ruigang Yang. Search Space Reduction for MRF Stereo. In *ECCV*, 2008.

[6] Qingxiong Yang, Liang Wang, and Narendra Ahuja. A constant-space belief propagation algorithm for stereo matching. In *CVPR*, 2010.

This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.