

Radial Distortion Homography

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The estimation of a homography between two views is a crucial problem in computer vision with many application, *e.g.*, in image stitching, structure from motion, or camera calibration. A homography exists between projections of points on a 3D plane in two views or between projections of general 3D points in two views when the transformation between the two views is only a pure rotation.

A number of algorithms have been proposed for homography estimation in the past. The classical linear algorithm [5] estimates the general homography from 4 point correspondences. This algorithm is frequently used in many applications, but it assumes perspective projection and thus it doesn't provide accurate estimates for image correspondences corrupted by radial distortion. This is a serious drawback, since virtually all real projections involve some amount of radial distortion. It was shown that ignoring the radial distortion, even for standard consumer cameras, may lead to significant errors in 3D reconstruction [4], metric measurements from images, or in camera calibration.

One way to deal with radial distortion is to model it in the final optimization step, *i.e.*, in the bundle adjustment. However, this approach requires a set of correct image matches which may be difficult to find and thus in many situations—and especially for larger radial distortions—it is necessary to consider the distortion already when searching for image correspondences in a RANSAC loop [3]. By not providing correct initial radial distortion estimates, the bundle adjustment is more likely to fall into a local minima without recovering correct distortion parameters.

The main motivation of this paper is to fill the gap among the homography estimation methods by developing an algorithm that can handle both planar and pure rotation scenes as well as cameras with different radial distortions. The goal is to propose an algorithm that uses as low a number of point correspondences as possible while being sufficiently fast to be used inside a RANSAC loop [3] in real-time applications or in the seed detection phase of large SfM pipelines [6].

We have developed two algorithms—solvers—for estimating unknown homography and radial distortion parameters of two cameras. The first solver, $H5\lambda_1\lambda_2$, uses the minimal number of five image point correspondences and solves a nonlinear system of polynomial equations using the Gröbner basis method. This solver performs Gauss-Jordan (G-J) elimination of quite a small 16×21 matrix and provides five solutions from the eigenvalues of a 5×5 matrix. The second solver, $H6\lambda_1\lambda_2$, uses six image point correspondences and solves a simple system of two quadratic equations in two unknowns and one system of six linear equations. This leads to two plausible solutions.

We consider different radial distortions for each camera and one-parameter division model for radial distortion modeling [4]. This model has the form:

$$f_u(\hat{\mathbf{x}}_i, \lambda) = \left[\hat{x}_i, \hat{y}_i, 1 + \lambda(\hat{x}_i^2 + \hat{y}_i^2) \right]^T, \quad (1)$$

where $\hat{\mathbf{x}}_i = [\hat{x}_i, \hat{y}_i, 1]^T$ are the homogeneous coordinates of the measured (and radially distorted) image points and $\lambda \in \mathbb{R}$ is the distortion parameter.

The two new proposed solvers solve the equation relating image correspondences distorted with different amounts of radial distortion

$$\alpha_i f_u(\hat{\mathbf{x}}'_i, \lambda') = H f_u(\hat{\mathbf{x}}_i, \lambda), \quad (2)$$

where α_i is an unknown scalar value and $H \in \mathbb{R}^{3 \times 3}$ is the unknown homography between the views.

Table 1 shows the comparison of all algorithms related to the two new proposed solvers w.r.t. the number of point correspondences used, the number of solutions, and other properties. It is clear that the only algorithms that work for both planar as well as pure rotations scenes and that can handle

	$F9\lambda_1\lambda_2$	H4	H3 λf	H5 λ	$H6\lambda_1\lambda_2$	$H5\lambda_1\lambda_2$
Reference	[1]	[5]	[2]	[4]	this	this
Plane		✓		✓	✓	✓
Pure rotation		✓		✓	✓	✓
Radial distortion	✓		✓	✓	✓	✓
Different distortions	✓				✓	✓
Minimal solution	✓	✓	(✓)	(✓)		✓
Number of points	9	4	3	5	6	5
Number of solutions	24	1	18 (25)	18	2	5

Table 1: Comparison of properties of various solvers.

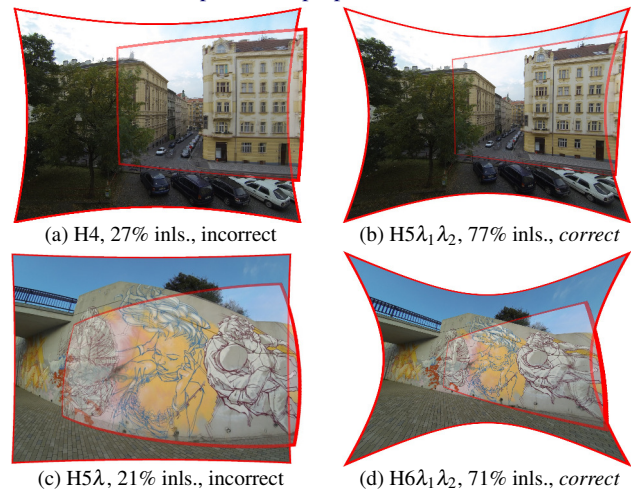


Figure 1: Stitching of (a-b) images connected by rotation induced homography initialized by H4, and $H5\lambda_1\lambda_2$, respectively, (c-d) images connected by plane induced homography initialized by H5 λ , and $H6\lambda_1\lambda_2$, respectively. The percentages show the ratio of the number of inliers found by LO-RANSAC for the respective solver to the number of tentative matches.

different radial distortions at the same time are the new solvers. Moreover, the new solvers return less solutions than their competitors ($F9\lambda_1\lambda_2$, H3 λf , H5 λ) while being faster and less complicated. These properties are crucial for solvers that are to be used in a RANSAC-style loop [3].

To precisely gauge the stability, precision, and the speed of the proposed solvers $H5\lambda_1\lambda_2$ and $H6\lambda_1\lambda_2$, we tested the solvers on synthetic as well as real world data with various amounts of radial distortions, noise levels, and scene configurations. We compared the proposed solvers with a comprehensive set of related algorithms— $F9\lambda_1\lambda_2$ [1], H4 [5], H3 λf [2], and H5 λ [4]. The experiments show that the proposed algorithms are stable, very fast ($\approx 2 \mu s$), and can be efficiently used inside a RANSAC loop. Figure 1 shows examples of image pairs connected by camera rotation induced homography (a-b) and by a plane induced homography (c-d) with different radial distortions λ_1, λ_2 stitched together using different solvers.

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