A Geodesic-Preserving Method for Image Warping

Dongping Li[1], Kaiming He[2], Jian Sun[2], Kun Zhou[1],
[1]Zhejiang University. [2]Microsoft Research.

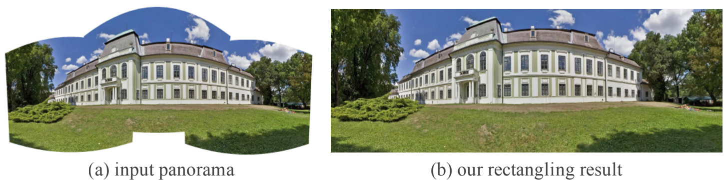

Figure 1: An example of our method. Given an input panorama with irregular boundaries (a), our method warps it to regular boundaries while keeps its (geodesic) appearance (b).

The manipulation of panoramic/wide-angle images is usually achieved via image warping. Existed methods [1, 2, 3, 4, 5] preserve straight lines besides shapes. They are not sufficient for warping panoramic/wide-angle images: image projections will turn straight lines into curved "geodesic lines", and it is fundamentally impossible to keep all these lines straight. Instead of preserving straightness, in this work we propose to preserve geodesic lines.

We define "geodesic lines" as projections of 3-D straight lines onto 2-D manifolds. Unlike the methods in [1, 5] that straighten geodesic lines, our method allows them to be curved. But an unnaturally curved geodesic line can be noticeable, because a geodesic line is not simply a locally smooth curve. In our solution, we constrain a geodesic line to remain "geodesic": it should be warped into another geodesic line, so can preserve its geodesic appearance. Fig. 1 is an example of our solution.

Given an input panoramic/wide-angle image, we first detect geodesic lines. We then group geodesic lines such that segments on the same plane are grouped together. The segments in the same group should have a non-local property, *i.e.*, they are expected to lie on a common plane after warping. This non-local property is given by the two rotation angles (θ,ϕ) that rotate one plane to another. Next we present a warping energy function that only involves the non-local variables (θ,ϕ) and the mesh vertexes.

Consider a single segment with two endpoints $\hat{\mathbf{p}}_1$, $\hat{\mathbf{p}}_2$ before warping (Fig. 2(a)). $\hat{\mathbf{p}}_1$, $\hat{\mathbf{p}}_2$ are 3-D points and represented as 3×1 vectors. We use the camera center as the origin of the 3-D coordinates.

Assume a 3-D point $\mathbf{p}$ can be modeled by a transform $\mathcal{T}$ from $\hat{\mathbf{p}}_1$, $\hat{\mathbf{p}}_2$. The transform involves two parts. In the first part, it is shifted inside the plane spanned by the two vectors $\hat{\mathbf{p}}_1$, $\hat{\mathbf{p}}_2$ (see $\hat{\mathbf{p}}$ in Fig. 2(b)). If we use a 3×2 matrix $\hat{\mathrm{B}}=\{\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2\}$ to denote the basis of this plane, then $\hat{\mathbf{p}}$ can be written as $\hat{\mathrm{B}}\mathbf{s}$, where $\mathbf{s}$ is a 2×1 vector to be determined. In the second part, the transform rotates this plane by some angles (θ,ϕ) (Fig. 2(b)). This rotation can be written as a 3-D rotation matrix $\mathrm{R}_{\theta,\phi}$. Combining these two parts, the transform $\mathcal{T}$ is: $\mathcal{T}(\mathbf{s},\theta,\phi)=\mathrm{R}_{\theta,\phi}\hat{\mathrm{B}}\mathbf{s}$. We define an energy to minimize the difference between a point $\mathbf{p}$ and its expected transformed position.

$$e(\mathbf{p},\mathbf{s},\theta,\phi)=\|\mathrm{R}_{\theta,\phi}\hat{\mathrm{B}}\mathbf{s}-\mathbf{p}\|^2. \tag{1}$$

Here $\mathbf{p}$ is the 3-D position after warping and will be related to the mesh vertexes, and $\{\theta,\phi\}$ are *non-local* variables that are shared by all the segments in the same group.

We first minimize Eqn.(1) w.r.t. $\mathbf{s}$ and obtain: $\mathbf{s}=(\hat{\mathrm{B}}^{\mathrm{T}}\hat{\mathrm{B}})^{-1}\hat{\mathrm{B}}^{\mathrm{T}}\mathrm{R}^{\mathrm{T}}_{\theta,\phi}\mathbf{p}$ This shows a nice property that $\mathbf{s}$ is a linear function of $\mathbf{p}$. Substituting $\mathbf{s}$ into Eqn.(1) we obtain:

$$e(\mathbf{p},\theta,\phi)=\|\mathrm{C}_{\theta,\phi}\mathbf{p}\|^2, \tag{2}$$

with a matrix $\mathrm{C}_{\theta,\phi}$ defined as: $\mathrm{C}_{\theta,\phi}\triangleq\mathrm{R}_{\theta,\phi}\hat{\mathrm{B}}(\hat{\mathrm{B}}^{\mathrm{T}}\hat{\mathrm{B}})^{-1}\hat{\mathrm{B}}^{\mathrm{T}}\mathrm{R}^{\mathrm{T}}_{\theta,\phi}-\mathrm{I}$.

This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.

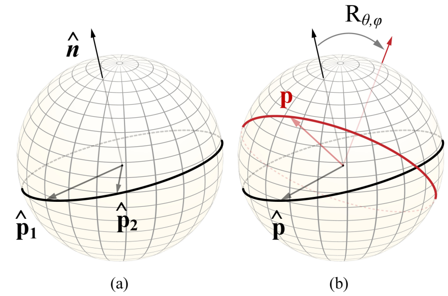

Figure 2: Left: $\hat{\mathbf{p}}_1$ and $\hat{\mathbf{p}}_2$ are two end points of a geodesic line segment before warping. Right: $\hat{\mathbf{p}}=\hat{\mathrm{B}}\mathbf{s}$ is a shifted point on the same plane (black) as $\hat{\mathbf{p}}_1$ and $\hat{\mathbf{p}}_2$. This plane is rotated by $\mathrm{R}_{\theta,\phi}$ and mapped to another plane (red). Then $\hat{\mathbf{p}}$ becomes $\mathbf{p}$ after warping.

Given all segments clustered into K groups, the energy E_G for preserving all geodesic lines is:

$$E_G(\{\mathbf{p}\},\{\theta_k,\phi_k\})=\frac{1}{L}\sum_{k=1}^{K}\sum_{l\in\mathcal{G}(k)}\sum_{i=1,2}\|\mathrm{C}_{\theta_k,\phi_k}\mathbf{p}_{l,i}\|^2. \tag{3}$$

Here L is the number of segments, $\mathbf{p}_{l,i}\ {}_{(i=1,2)}$ are the two end points in a segment l, and l belongs to the k-th group $\mathcal{G}(k)$. The notations θ_k and ϕ_k imply that the rotation angles are shared by the segments in the group k, such that these segments are expected on the same plane after warping. So $\{\theta_k,\phi_k\}$ are non-local variables of the group k.

Next we incorporate the geodesic-preserving energy Eqn.(3) in a warping energy. We consider quad meshes in this paper. The vertexes are denoted as $\{\mathbf{v}_j\}$ with each $\mathbf{v}_j=(u_j,v_j)$ as 2-D coordinates. Denote all the vertexes by a vector $\mathbf{V}$. The warping energy is defined as:

$$E(\mathbf{V},\{\theta_k,\phi_k\})=\lambda_B E_B(\mathbf{V})+\lambda_S E_S(\mathbf{V})+\lambda_G E_G(\mathbf{V},\{\theta_k,\phi_k\}). \tag{4}$$

Here E_B is a boundary-preserving term, E_S is a shape-preserving term, and E_G is the geodesic-preserving term defined on vertices. We set $\lambda_B=10^8$ for hard constraints, and set $\lambda_S=1$ and $\lambda_G=100$. To optimize this energy, we adopt an alternative scheme between $\mathbf{V}$ and $\{\theta_k,\phi_k\}$. The details of the terms and optimization are in the paper.

Our method is demonstrated in various applications, including rectangling panoramas, resizing panoramic/wide-angle images, and wide-angle image manipulation. An extension to ellipse preservation for general images is also presented.

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