Heat Diffusion Over Weighted Manifolds: A New Descriptor for Textured 3D Non-Rigid Shapes

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This paper proposes an approach for modeling textured 3D non-rigid models based on Weighted Heat Kernel Signature(W-HKS). As a first contribution, we show how to include photometric information as a weight over the shape manifold, we also propose a novel formulation for heat diffusion over weighted manifolds. As a second contribution we present a new discretization method for the proposed equation using finite element approximation. Finally, the weighted heat kernel signature is used as a shape descriptor. The proposed descriptor encodes both the photometric, and geometric information based on the solution of one equation. We also propose a new method to introduce the scale invariance for the weighted heat kernel signature. The performance is tested on two benchmark datasets. The results have indeed confirmed the high performance of the proposed approach.

In the past decade, significant effort has been invested in extending the invariance properties to non-rigid deformations [3]. All these efforts have focused only on the 3D shape. Recently, taking the photometric information into account to calculate a 3D shape descriptor has attracted more research. The work of Kovnatsky et al. [5] uses the diffusion geometry framework for the fusion of geometric and photometric information. Their construction is based on an *ad hoc* definition of a diffusion process. S. Biasotti et al. [1] proposed the PHOG descriptor as a combination of photometric, hybrid and geometric descriptions into one descriptor for textured 3D object retrieval.

In this paper, we develop, for the first time, a mathematical framework for the diffusion geometry on textured shapes. We present an approach for shape matching and retrieval based on weighted heat kernel signature.

A weighted manifold (called also a manifold with density)is a Riemannian manifold $\mathcal{M}$ endowed with a measure μ that has a smooth positive density h with respect to the Riemannian measure σ. The weighted Laplace operator $\triangle_{\mathcal{M},\mu}$, generalizing the Laplace-Beltrami operator, is defined by $\triangle_{\mathcal{M},\mu}U = div_{\mu}\nabla U$ for any smooth function U on $\mathcal{M}$.

then weighted heat equation can be written as:

$$\triangle_{\mathcal{M},\mu}U(\mathbf{x},t) = -\frac{1}{2}\frac{\partial}{\partial t}U(\mathbf{x},t) \quad (1)$$

with initial condition $U(\mathbf{x},0) = u(\mathbf{x})$ and Dirichlet boundary condition $U(\mathbf{x},t) = 0$ for all $\mathbf{x} \in \partial\mathcal{M}$ and all $t > 0$.

The weak formulation of the weighted heat equation (1) is obtained by multiplying by a test function $\varphi \in \mathcal{C}^2$ and integrating the resulting relation over the weighted manifold $(\mathcal{M},\mu)$

$$\int_{\mathcal{M}} \varphi \frac{\partial}{\partial t}U(\mathbf{x},t)d\mu + \frac{1}{2}\int_{\mathcal{M}} \varphi \triangle_{\mathcal{M},\mu}U(\mathbf{x},t)d\mu = 0 \quad (2)$$

where $d\mu = hd\sigma$, $d\sigma$ is the Riemannian measure and h is a smooth positive density. Then the weighted heat kernel signature will be

$$K(\mathbf{x},\mathbf{x},t) := B\phi^2 diag(D(t)) \quad (3)$$

where $B,\phi,D(t)$ as defined in the paper.

We propose to construct the descriptor as follows: The W-HKS descriptor is calculated for each triangle mesh based on Eq. 3. We calculated the W-HKS at all points of a shape over the three normalized color channels (RGB). The color information is considered as the weight h at each vertex. W-HKS is calculated at different time scales. Then the scale normalization step is applied as explained in the paper. Considering only the first 20 significant FT components the size of the descriptor now will be $n \times 20$ for n vertex shape. Then we used the Bag of Features (BoF) to represent the shape as one feature vector. The bags of features were created using vocabulary of size 64. Thus the feature vector size is 64×3 for any 3D shape.

This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.

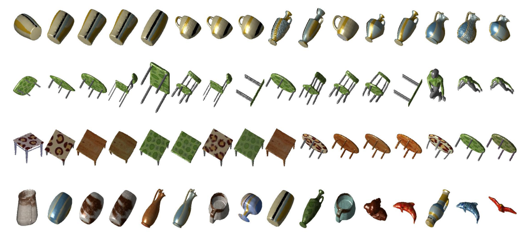

Figure 1: Shape retrieval results of: SHREC'13 dataset in the upper two rows, SHREC'14 dataset in the lower two rows. Left: queries. Right: First 15 matches using the W-HKS2 descriptor.

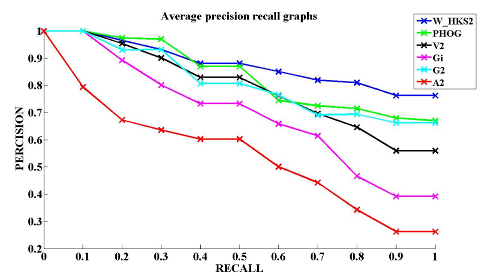

Figure 2: Shape retrieval results of SHREC'13 dataset. Precision-recall curves for all methods.

To test the performance of the proposed approach we use the SHREC'13 dataset [4] and the SHREC'14 dataset [2]. We compare our method with the four best methods in the retrieval competition on textured 3D models [4], and with the PHOG approach proposed in [1]. Fig. 1 shows some retrieval results in the SHREC'13 dataset, and the SHREC'14 dataset. Fig. 2 shows the performances of all methods in terms of average precision-recall curves in the SHREC'13 dataset.

This paper has addressed the problem of textured 3D shapes representation. We have presented a new approach for shape matching and retrieval based on Weighted Heat Kernel Signature (W-HKS). We proposed to use the color information as a weight over the shape manifold. We also proposed a novel formulation for heat diffusion over weighted manifolds. Then we presented a new discretization method for the weighted heat kernel based on FEM. Finally, the weighted heat kernel signature is used as a shape descriptor. Our experimental results have shown that the proposed descriptor can achieve high performance on SHREC'13 and SHREC'14 benchmark datasets. The proposed approach has outperformed state-of-the-art approaches (five different methods) for textured shapes representation and retrieval. Different evaluation measures approved the high accuracy of the proposed framework.

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