Subgraph Matching using Compactness Prior for Robust Feature Correspodence

Yumin Suh, Kamil Adamczewski, Kyoung Mu Lee
Department of ECE, ASRI, Seoul National University, Seoul, Korea

Feature correspondence is widely formulated as a graph matching problem due to its robust performance under challenging conditions. A variety of fast and accurate algorithms have been proposed for graph matching. However, most of them focus on improving the recall of the solution while rarely considering its precision, thus inducing a solution with numerous outliers [1, 7]. To address both precision and recall feature correspondence should rather be formulated as a subgraph matching problem. This paper proposes a new subgraph matching formulation which uses a compactness prior, an additional prior that adjusts the number of matches and effectively eliminates outliers. To solve the new optimization problem, we propose a meta-algorithm based on Markov chain Monte Carlo [3]. By constructing Markov chain on the restricted search space instead of the original solution space, our method approximates the solution effectively. The experiments indicate that our proposed formulation and algorithm significantly improve the baseline performance under challenging conditions when both outliers and deformation noise are present.

In the conventional integer quadratic program (IQP) formulation [1, 4] used for graph matching, the quality of a matching between two graphs is measured as the sum of affinity values between all pairs of nodes and edges given by the correspondence. It is formulated as follows:

$$\begin{aligned} &\mathbf{y}^* = \arg\max_{\mathbf{y}} \mathbf{y}^T \mathbf{M} \mathbf{y} \\ &s.t. \quad \mathbf{y} \in C_{int} \cap C_{one}. \end{aligned} \tag{1}$$

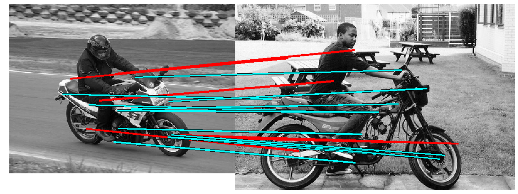
$\mathbf{y} \in \{0,1\}^{n_1 n_2}$ is a binary indicator vector of $n_1 n_2$ possible node correspondences, where n_1 and n_2 are the number of nodes in each graph, and $\mathbf{M} \in \mathbf{R}^{n_1 n_2 \times n_1 n_2}$ is an *affinity matrix* which consists of affinity values, indicating compatibility between i^{th} and j^{th} possible match in its (i, j)-component. $C_{int} = \{\mathbf{y} | \mathbf{y} \in \{0,1\}^{n_1 n_2}\}$ enforces the integer constraint and $C_{one} = \{\mathbf{y} | \forall i, \sum_a \mathbf{y}_{ia} \leq 1\} \cap \{\mathbf{y} | \forall a, \sum_i \mathbf{y}_{ia} \leq 1\}$ enforces the one-to-one constraint. The affinity measure is typically restricted to be nonnegative, therefore the number of matches in the final solution is implicitly determined to be the maximum possible number of matches under one-to-one constraint, that is $min(n_1, n_2)$ because increasing the number of matches results in higher objective. The underlying assumption is that the solutions that contain true matches have higher objective score than those do not. However, this assumption poorly holds even with few outliers (Section 3.1). As a result, optimizing (1) may not necessarily result in the detection of the true matches.

To overcome the problem, we propose to augment the original objective of the IQP with an additional compactness prior as follows:

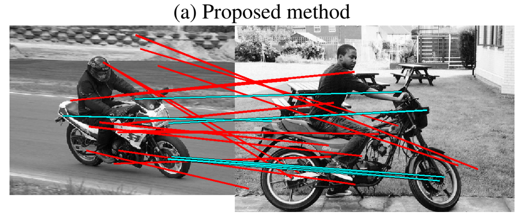
$$\begin{aligned} &\mathbf{y}^* = \arg\max_{\mathbf{y}} \mathbf{y}^T \mathbf{M} \mathbf{y} - \lambda^T \Omega(\|\mathbf{y}\|) \\ &s.t. \quad \mathbf{y} \in C_{int} \cap C_{one}, \end{aligned} \tag{2}$$

where $\Omega(\|\mathbf{y}\|) = [\omega_1(\|\mathbf{y}\|), \omega_2(\|\mathbf{y}\|), \cdots, \omega_n(\|\mathbf{y}\|)]$ are the penalties, and $\lambda = [\lambda_1, \cdots, \lambda_n]^T$ is the parameter vector that adjusts the weight of each component $\omega_i(\|\mathbf{y}\|)$. In this paper, we used $\Omega(\|\mathbf{y}\|) = [\|\mathbf{y}\|, \|\mathbf{y}\|^2]$ for simplicity. The objective function enforces the matches to have high total affinity by the first term, while adjusting the number of matches in the solution by the second term. The level of compactness is determined by the parameter λ, whose value depends both on the affinity measure and the graph size. The proposed formulation (2) is a generalized form of (1) which becomes the original one when $\lambda = \mathbf{0}$.

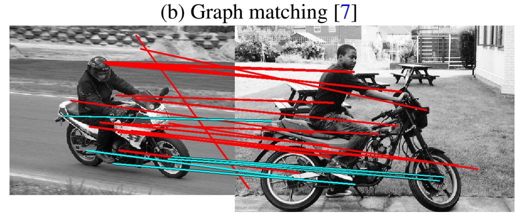
For a given problem setting, the best parameter λ is trained to maximize the margin. We train the parameter using the structuerd support vector machine [5], as in [2]. Training the parameter λ ensures the augmented prior in the proposed formulation (2) to be at least not harmful. If there is no non-zero λ ($\lambda \neq \mathbf{0}$) that further reduces the empirical loss, then λ is learned to be $\mathbf{0}$, while making the (2) equivalent to the original IQP of (1).



(a) Proposed method



(b) Graph matching [7]



(c) Graph matching [1]

Figure 1: An example of feature correspondence. True positives matches are represented by blue lines and false positive matches are represented by red lines.

To solve the formulation (2), we propose an algorithm based on Markov chain Monte Carlo (MCMC) sampling technique.The algorithm effectively approximates the solution by iteratively selecting the subset of all nodes to be matched and the finding the matches given the selected nodes.

The experimental results show that the proposed method outperforms or is comparable to state-of-the-art methods [6, 7] in terms of both precision and recall. In feature correspondence problem, the proposed method performs robustly in the noisy environment and in the presence of outliers.

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This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.