# GMMCP Tracker: Globally Optimal Generalized Maximum Multi Clique Problem for Multiple Object Tracking

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Data association is the backbone to many multiple object tracking (MOT) methods. Reviewing the literature, most data association methods have considered a simplified version of the problem and focused on approximate inference methods which can be solved efficiently[2, 3]. On the other side, those algorithms which incorporate more accurate formulation of tracking scenario in real world suffer from greedy optimizations and local minima [1, 4]. In this paper we take a different direction and propose to formulate tracking as a new graph theoretic problem called *Generalized Maximum Multi Clique Problem* (GMMCP). Our tracking formulation does not involve any simplification in problem formulation nor in optimization. Our method has several advantages: First, it mimics the real world tracking scenario precisely by incorporating all temporal pairwise relationship in a batch of frames. Second, we formulate the proposed graph theoretic problem using Binary-integer Program without simplifying the original problem. Third, it allows including higher order relationship between targets in our cost function. Fourth, it can robustly handle short term and long-term occlusion. And, finally it lends itself to real-time implementation on a desktop computer.

**Generalized Maximum Multi Clique Problem.** Given a k-partite complete graph, where there is an edge between every pair of nodes that do not belong to the same cluster; the objective is to find a sub-graph which forms $K$ cliques, in which the sum of edges of the cliques are maximized and exactly $K$ nodes are selected from every cluster. To give a more formal definition, the input to the GMMCP is a graph $G(V,E,W)$, where $V$, $E$ and $W$ are the nodes, edges and their corresponding weights. $V$ is divided into a set of disjoint clusters where $v_i^j$ defines the $i^{th}$ node in the $j^{th}$ cluster. The goal is now to pick a set of $K$ cliques by selecting exactly $K$ nodes from each cluster that maximize the total score.
We formulate GMMCP as a Binary Integer Program. To the best our knowledge this problem is not solved before. Any binary integer program can be formulated as follows:

$$\begin{cases} \text{maximize} & \mathbf{C}^T\mathbf{x}, \\ \text{subject to} & \mathbf{Ax} = \mathbf{b} \text{ and } \mathbf{Mx} \leq \mathbf{n}. \end{cases} \tag{1}$$

The objective function to maximize is $\mathbf{C}^T\mathbf{x}$, where $\mathbf{C}$ is the weight matrix and $\mathbf{x}$ is boolean column vector. $\mathbf{Ax} = \mathbf{b}$ and $\mathbf{Mx} \leq \mathbf{n}$ define the equality and inequality constraints. For every node and edge in the graph there is binary variable in vector $\mathbf{x}$. Let us define the $v_i^j$ as the binary variable for each node (the $i^{th}$ node in the $j^{th}$ cluster) and $e_{ij}^{mn}$ to be the binary variable for the edge between the nodes $v_i^j$ and $v_m^n$. In order to make sure that the solution found by $\mathbf{x}$ is a feasible solution to GMMCP, $\mathbf{x}$ needs to satisfy three constraints.
The first **constraint** enforces the sum of nodes in each cluster to be equal to $K$, which is the number cliques we need to find.

$$\{\forall j | 1 \leq j \leq h\} : \sum_{i=1}^{l} v_i^j = K, \tag{2}$$

where $h$ is the number of clusters and $l$ is the number of nodes within that cluster. The second **constraint** ensures that, if a node is selected then $(h-1)$ of its edges should be included in the solution. This is because of the fact that in each clique, one node from each cluster is included.

$$\sum_{j=1}^{h}\sum_{i=1}^{l} e_{mn}^{ij} = v_m^n.(h-1).\{\forall m,n | 1 \leq n \leq h, 1 \leq m \leq l\} \tag{3}$$

Finally we need the third **constraint** to ensure that the solution found by $\mathbf{x}$

This is an extended abstract. The full paper is available at the Computer Vision Foundation web page.

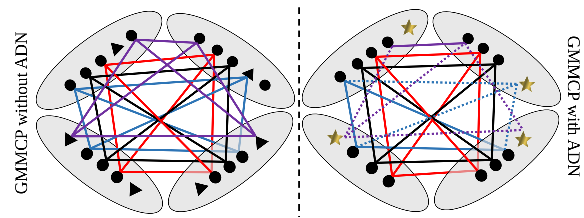


Figure 1: The two versions of the input graph used in our tracking. The graph on the left uses regular dummy nodes (shown with triangles) for occlusion handling and the speed-up version of our tracker uses only one dummy node per cluster called aggregated-dummy-node (shown with stars). Our method finds all the cliques in a graph that maximize the score function simultaneously. In this example 4 cliques are found, each shown in a different color. Each circle represents one tracklet.

will form a clique.

$$e_{ij}^{i'j'} + e_{i'j'}^{i''j''} \leq 1 + e_{i''j''}^{ij}. \quad \{\forall e_{ij}^{mn} \in E\} \tag{4}$$

The combination of these three constraints will ensure that $\mathbf{x}$ will provide a valid solution to the GMMCP. Once $\mathbf{x}$ is found, each clique will represent a track of a person.

**Occlusion Handling.** In our formulation each node will represent a tracklet of person which may not necessarily be present in all the frames (cluster) or may be occluded or miss-detected. In order to avoid selecting irrelevant nodes in a track of a person, we add an additional set of nodes to each cluster called *dummy nodes*. Dummy nodes are treated the same as the rest of the nodes in the graph with only one difference. The weights of the edges connected to each dummy node are fixed to a pre-defined value of $c_d$. Our dummy nodes will ensure that the tracks for each person will be free of outliers. In other words, when there is no confident detection for a clique in a particular cluster, a dummy node from that cluster is selected.

**Speed-Up.** In the paper we show that one can easily obtain the upper bound for the number of dummy nodes added to each cluster. However, in practice only a small number of these dummy nodes are sufficient to handle miss detections in cliques. Adding more dummy nodes will increase the computational complexity as the number of variables during optimization will increase. In order to void such cases, we introduce *Aggregated Dummy Nodes (ADN)*. Our ADN will no longer be boolean variable and can take any integer value. This allows us to add only one ADN to each cluster which will account for all the outliers in that cluster. Figure 1 shows the two version of the input graphs used in our method. ADNs will require a different set of constraints to ensure the solution is a valid solution to GMMCP. In order to satisfy the new set of constraints (explained in the paper) we propose a Mixed-Binary Integer Programing solution and show that the new formulation lends itself to a realtime implementation on a desktop computer.

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