A Convex Optimization Approach to Robust Fundamental Matrix Estimation

Y. Cheng ,J. A. Lopez, O. Camps, M. Sznaier
Electrical and Computer Engineering, Northeastern University, Boston, MA 02115.

Given a pair of images of the same scene from two uncalibrated perspective views, the fundamental matrix $\mathbf{F} \in \mathbb{R}^{3\times 3}$ is defined as the rank-2 matrix satisfying the epipolar constraint

$$\mathbf{x}'^T \mathbf{F}\mathbf{x} = 0, \forall \mathbf{x}', \mathbf{x}$$

where the homogenous coordinates $\mathbf{x}, \mathbf{x}' \in \mathbb{R}^3$ are the corresponding projections of a 3D point in the two images. Given the key role that $\mathbf{F}$ plays in a large number of applications, the problem of estimating it from experimental data has been the subject of a large research effort. Existing techniques handle the non-convex rank-2 constraint either by (i) using a two-step approach [2], that starts by finding an unconstrained (sub)optimal estimate, and then refining it by reducing its smallest singular value to 0, or (ii) directly incorporating it into a non-convex optimization [1]. Although the above methods perform well in low noise scenarios, performance degrades substantially in the presence of even a few point mismatches. Alternatively, randomized methods (see e.g. [5] and references therein) attempt to find outlier-free data by repeatedly randomly selecting the minimal number of correspondences needed to generate a solution, and selecting the best one, according to some optimality criteria. These approaches are attractive due to their simplicity and the availability of bounds on the number of iterations required to guarantee a given probability of success. However, these bounds grow very fast with the number of outliers. Secondly, since the bounds explicitly depend on the number of outliers, this quantity must be known or estimated accurately, since stopping the algorithm prematurely can lead to arbitrarily bad solutions. Finally, these methods cannot directly impose the rank deficiency constraint. Rather, this is done a posteriori, by projecting the solution onto the manifold of rank-2 matrices. However, this step can lead to substantial performance degradation [4].

Motivated by the challenges noted above, in this paper we propose a single-step framework for robustly estimating the fundamental matrix from point correspondences corrupted by noise and outliers. Specifically, we address the following problem:
P1: *Given a set of n noisy point correspondences, $\{\mathbf{x}_i, \mathbf{x}'_i\}$, drawn from two images of the same scene, and a-priori bound on the fitting error $|\mathbf{x}_i'^T \mathbf{F}\mathbf{x}_i| \leq \varepsilon$, find a matrix $\mathbf{F}$ such that (i) $||\mathbf{F}||_F = 1$; (ii) rank($\mathbf{F}$)=2; and (iii) The number of inliers is maximized.*

The main idea of the paper is to introduce binary variables $s_i \in \{0,1\}$ that indicate whether a given correspondence is an inlier, and to impose the rank-2 constraint by searching for the epipoles, leading to the following polynomial optimization:

$$p^* = \min_{\mathbf{q},\mathbf{F},s_i} \sum_{i=1}^{n} (1-s_i) \text{ subject to:} \quad (1)$$

$$||\mathbf{F}||_F^2 = 1,\ \mathbf{F}\mathbf{q} = 0, \mathbf{q}^T\mathbf{q} = 1, \quad (2a)$$

$$s_i|\mathbf{x}_i'^T \mathbf{F}\mathbf{x}| \leq s_i\varepsilon, \quad (2b)$$

$$s_i^2 = s_i, \forall_{i=1}^n \quad (2c)$$

Here (2c) forces s_i to be binary, and combined with (2b) enforces that $s_i = 0$ for outliers. Thus, the objective function (1) is indeed the number of outliers.

Our main result shows that the problem above can be efficiently solved by exploiting sparse polynomial optimization techniques [3]. Specifically, the advantages of the proposed approach vis-à-vis existing techniques, illustrated in Fig. 1, include the abilities to:

- Explicitly impose rank-deficiency and handle noise and a very large percentage of outliers, without the need for assuming bounds on the number of outliers.
- Exploit co-occurrence priors to improve the estimate.
- Handle partially known correspondences.

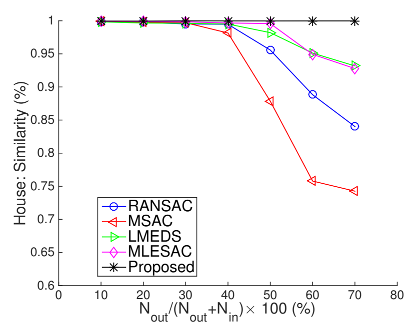

Figure 1: Trace($\mathbf{F}^T\mathbf{F}_{true}$) as a function of the % of outliers

This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.

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