Absolute Pose for Cameras Under Flat Refractive Interfaces

Sebastian Haner, Kalle Åström
Centre for Mathematical Sciences, Lund University

Refractive structure-and-motion problems come in many varieties, depending on what relationships between scene structure, cameras and refractive interfaces are known. This paper studies the problem of determining the absolute pose of a camera observing known structure through a flat refractive interface, the location of which is known in the world coordinate system.

Refraction of light at an optical medium boundary is described by Snell's law which states that

$$\rho_1 \sin\theta_1 = \rho_2 \sin\theta_2 \,, \tag{1}$$

where $\rho_{1,2}$ are the refractive indices of the two media and $\theta_{1,2}$ the angles the impinging and refracted ray make with the surface normal. The impinging

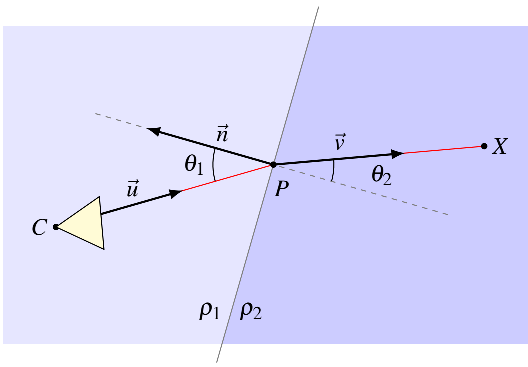

Figure 1: The image ray $(C,\vec{u})$ from the camera center intersects the plane with normal $\vec{n}$ at P and is refracted into the ray $(P,\vec{v})$ according to Snell's law.

ray with direction vector $\vec{u}$, the refracted ray $\vec{v}$ and the plane normal $\vec{n}$ must also all lie in the same plane. Using properties of the cross product, Snell's law may then be expressed on vector form as

$$\rho_1 \frac{\vec{u}\times\vec{n}}{\|\vec{u}\|\|\vec{n}\|} = \rho_2 \frac{\vec{v}\times\vec{n}}{\|\vec{v}\|\|\vec{n}\|} \tag{2}$$

or equivalently

$$r\|\vec{v}\|(\vec{u}\times\vec{n}) = \|\vec{u}\|(\vec{v}\times\vec{n}) \,, \tag{3}$$

where $r = \rho_1/\rho_2$. By squaring both sides component-wise we obtain three equations which are polynomial in all variables, but since both sides of (3) are orthogonal to $\vec{n}$ only two of them can be independent. The coplanarity constraint on the rays and normal can also be written as $\vec{u}\times\vec{v}\cdot\vec{n} = 0$, independently of the refractive indices. It is obvious that the camera center C and scene point X must also lie in this plane, implying

$$\vec{u}\times(X-C)\cdot\vec{n} = 0 \,, \tag{4}$$

which is also polynomial in all variables. Equations (3) and (4) can now be used together to derive minimal and near-minimal solvers for different variants of the absolute refractive pose problem. Using action matrix and polynomial eigenvalue problem techniques from algebraic geometry, solvers are derived for the following cases:

- Minimal solution in 2D using three point correspondences
- Minimal closed form solution with known camera orientation using two points
- Minimal solution using two points and known rotation axis, e.g. when an accelerometer provides the gravity vector in the camera coordinate system
- Near-minimal solution to the general calibrated pose problem using five points
- Near-minimal solution to the general problem with unknown focal length using six points.

The solvers are shown to be accurate and numerically stable, and the different variants run in between 15 and 80 ms as Matlab implementations. The solvers are further validated in experiments using real data and it is shown how neglecting refraction effects can lead to large errors.

We also note the large and quickly growing number of solutions to the derived polynomial systems, and attribute this to the fact that Snell's law as stated in (1) is ambiguous; it only specifies the angle the refracted ray makes with the normal, but not on which side, nor that the two rays should be on different sides of the plane (see Figure 2). This ambiguity gives rise to a

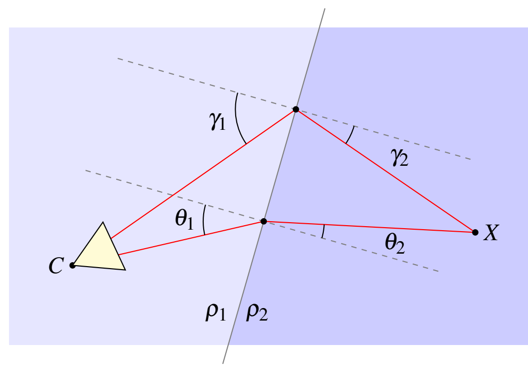

Figure 2: Ambiguity in Snell's law giving rise to false solutions. Both $\rho_1 \sin\theta_1 = \rho_2 \sin\theta_2$ and $\rho_1 \sin\gamma_1 = \rho_2 \sin\gamma_2$ are fulfilled.

large number of physically incorrect solutions which nevertheless satisfy the equations. The resulting explosion in the size of the equation systems for the unrestricted pose problem makes truly minimal solutions infeasible using this approach. The presented techniques are nevertheless flexible and should lend themselves well to other refractive problems. Source code for all solvers is available online at http://github.com/sebhaner/refractive_pose.

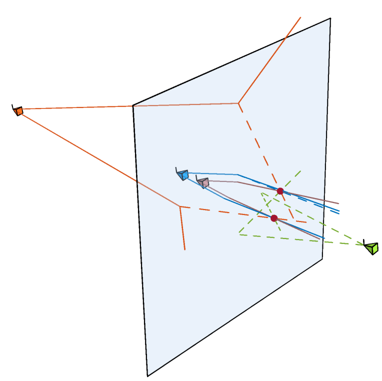

Figure 3: Four solutions to the known orientation, unknown refractive index case, three of which are incorrect due to ambiguities in the equations. Solid lines are the physical back-projections of the image points while dashed lines illustrate the spurious optical paths consistent with (3).

This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.