Maximum Persistency via Iterative Relaxed Inference with Graphical Models

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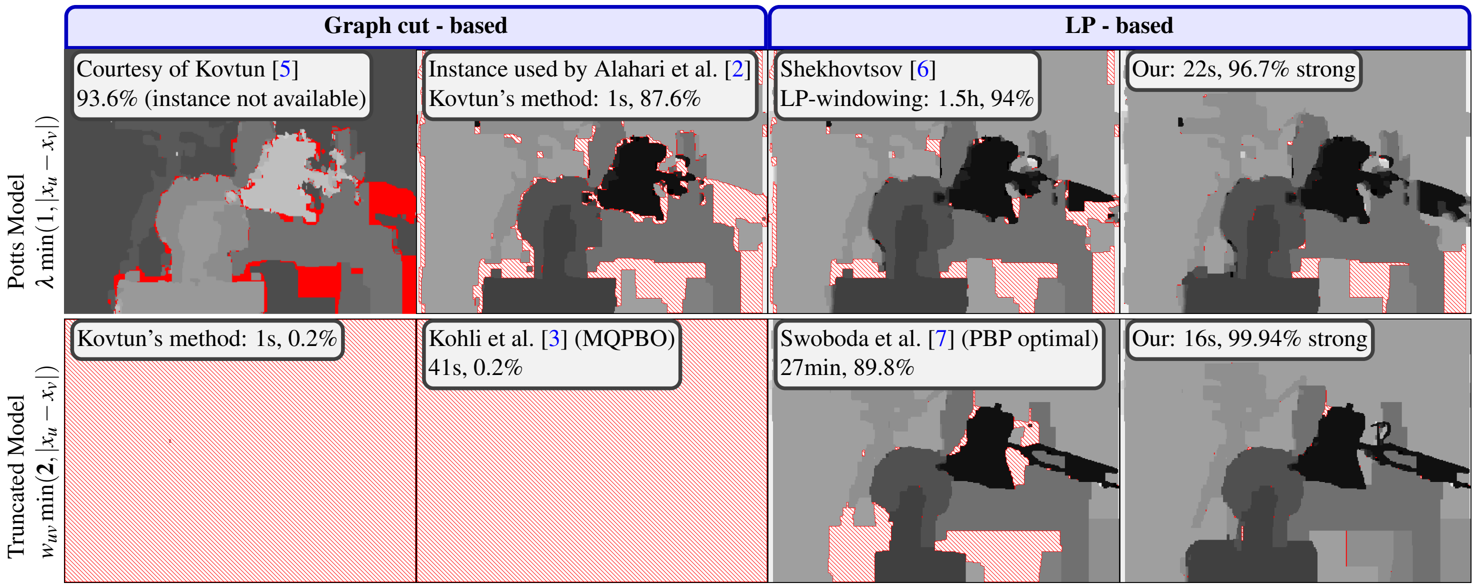

Figure 1: Progress of partial optimality methods. Top line corresponds to a stereo model with Potts interactions and large aggregating windows for unary costs used in [2, 5] (instance published by [2]). Bottom line is a more refined stereo model with truncated linear terms [8] (instance in [1]). Hashed area indicates that the optimal persistent label in the pixel is not found (but some non-optimal labels might have been eliminated). Solution completeness is given by the percent of persistent labels. Graph cut based methods are fast but only efficient for strong unary terms. LP-based methods are able to determine a larger persistent assignments but are extremely slow, prior to this work. Note, our method is set up to determine strong persistency, a partial assignment that holds for all optimal solutions, while other methods here find a part of any optimal solution.

We consider the NP-hard problem of MAP-inference for graphical models. We propose a polynomial time practically efficient algorithm for finding a part of its optimal solution. Specifically, our algorithm marks each label in each node of the considered graphical model either as (i) *optimal*, meaning that it belongs to all optimal solutions of the inference problem; (ii) *non-optimal* if it provably does not belong to any solution; or (iii) *undefined*, which means our algorithm can not make a decision regarding the label. The labels that we proved optimal or non-optimal are called *persistent*.

Key ideas:

- We build on the Maximum Persistency [6] framework, which proved that most of the existing methods for partial optimality can be explained by a simple local domination condition if only one supplies the right reparametrization of the energy function.
- Finding the maximum subset of persistent labels can be formulated [6] as a big linear program that optimizes over reparametrizations and a subset of labels deemed persistent at the same time. It is a challenging problem and large scale instances can only be addressed by a windowing technique [6] – a semi-local condition.
- We solve the same maximum persistency problem instead by iteratively solving standard LP relaxation for a series of auxiliary energy problems, similarly to the approach in [7]. We thus unite [6] and [7].

Key features of our approach:

- Invariant to reparametrization and order of labels.
- Fast approximate dual solvers can be employed without compromising correctness and global persistency guarantees.
- Requires an approximate solution to LP relaxation as a starting point.
- Can be viewed as making an approximate solver for LP-relaxation to be able to prove optimality of a part of its solution.

More specifically, we demonstrated our approach using TRW-S [4] for solving auxiliary subproblems.

Properties when subproblems are solved with TRW-S:

- Closely approximates maximum persistency LP (evaluated on small random problems).
- Fast message passing transfers to auxiliary problems.
- The method is correct using a finite number of TRW-S iterations.
- Subproblems can be solved incrementally, reusing the messages.

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