

# Dual-Space Decomposition of 2D Complex Shapes \*

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## Abstract

While techniques that segment shapes into visually meaningful parts have generated impressive results, these techniques also have only focused on relatively simple shapes, such as those composed of a single object either without holes or with few simple holes. In many applications, shapes created from images can contain many overlapping objects and holes. These holes may come from sensor noise, may have important parts of the shape or may be arbitrarily complex. These complexities that appear in real-world 2D shapes can pose grand challenges to the existing part segmentation methods. In this paper, we propose a new decomposition method, called Dual-space Decomposition that handles complex 2D shapes by recognizing the importance of holes and classifying holes as either topological noise or structurally important features. Our method creates a nearly convex decomposition of a given shape by segmenting both the polygon itself and its complementary. We compare our results to segmentation produced by non-expert human subjects. Based on two evaluation methods, we show that this new decomposition method creates statistically similar to those produced by human subjects.

## 1. Introduction

Decomposing two-dimensional (2D) shapes into functional and visually meaningful parts is a fundamental process in human vision [12, 15, 4, 5, 1, 30]. Many segmentation and decomposition techniques have been proposed to mimic this process computationally [6, 16, 31, 2]. While these techniques (usually known as *part segmentation*) generate impressive results, they also have only focused on relatively simple shapes, such as those composed of a single object either without holes [30] or with few simple holes [22, 25, 29]. Even when holes are explicitly considered, existing techniques usually decompose the shape so that no holes are left [22, 25, 29]. In many situations, shapes are

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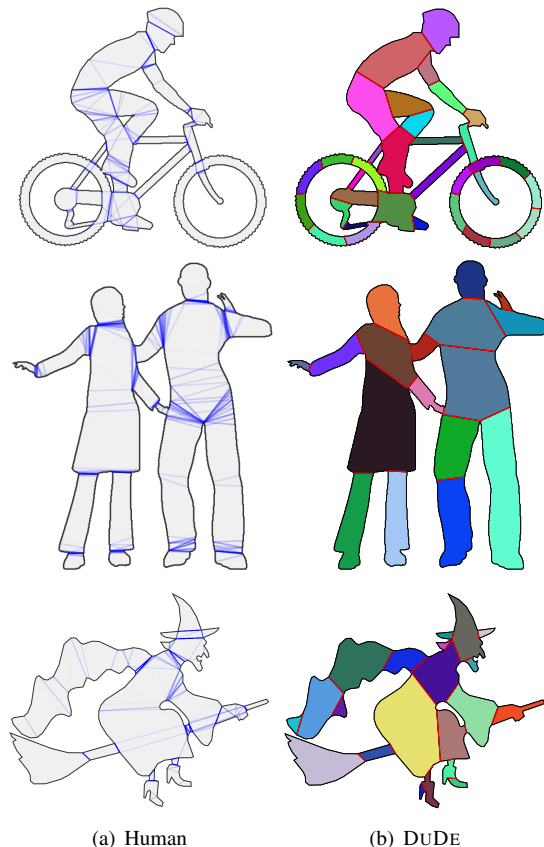


Figure 1. Examples of decompositions from human and dual-space decomposition (DUDE).

created from images (e.g., silhouettes) of overlapping objects and can also contain many undesired holes due to sensor noise. In other cases, these holes can be important part of the shape and arbitrary complex. These complex shapes, such as those shown in Figures 1, 6, and 7 pose grand challenges to the existing part segmentation methods.

Another major limitation in the existing part segmentation methods is the lack of well-defined criteria and benchmark for quality comparisons. Unlike image segmentation, comparison between part segmentation methods is usually done visually and evaluated based on the size of the final decompositions, i.e. smaller decomposition is better.

The most widely used MPEG 7 image set [21] consists of only shapes with single boundary. This limitation is further hindered by the lack of public domain implementations of many existing methods.

To address the aforementioned issues, in this paper, we propose a new decomposition method, called *Dual-space Decomposition* (or simply DUDE). DUDE is designed to handle complex 2D shapes that may be composed of overlapping objects with significant number of holes. DUDE accomplishes this by recognizing the importance of these holes by classifying holes as either topological noise and structurally important features. DUDE is developed based on the idea of nearly convex parts. Strategies that decompose shapes into nearly convex components have been proposed recently in the communities such as computer vision, pattern recognition and graphics [22, 33, 25, 29]. In order to recognize the importance features from the external and hole boundaries, DUDE creates the nearly convex decomposition of the shapes by segmenting both positive and negative regions of the shape (the polygon itself and its complementary).

In this paper, we propose an efficient dual-space method to decompose a non-convex polygon into several approximate convex parts, similar to human decomposition result which will be explained in following sections. Our method is designed to handle shapes with complex holes. To quantitatively evaluate DUDE, we take an initial step to collected more than 3800 segmentation from 142 non-expert using 72 polygons (shown in Figure 6). Using two statistics-based evaluation methods described in Section 5, we show that DUDE generates segmentation closer to man-made segmentation than existing methods.

## 2. Related Work

Polygon decomposition has been extensively studied theoretically and experimentally in areas including computer graphics, computational geometry [19], computer vision and pattern recognition.

In computational geometry, researchers are traditionally interested in creating decompositions subject to some optimization criteria, such as a minimum number of convex components [7, 13, 18, 8, 20]. Most of these problems are shown to be NP-hard [23, 18, 24]. More recently, several methods have been proposed to partition at salient features of a polygon. Tănase and Veltkamp [32] decompose a polygon based on the events that occur during the construction of a straight-line skeleton. Dey et al. [11] partition a polygon into collections of Delaunay triangles of points sampled from the polygon boundary. Lien and Amato [22] partition a polygon into *approximately convex* components. Their method reveals significant shape structures by recursively resolving the most concave features until the concavity of every component is below some user speci-

fied threshold. Wan [33] extends [22] to incorporate both concavity and curvatures and prevent over segmentation by avoid cuts inside pockets. Liu et al. [26] proposed the idea of  $\alpha$ -decomposition that use persistence analysis of features obtained from the continuous convolution [3] between the polygon and a disc.

In pattern recognition and computer vision, shape decomposition is usually a step toward shape recognition. For instance, Siddiqi and Kimia [30] use curvature and region information to identify *limbs* and *necks* of a polygon and use them to perform decomposition. Recently, Liu et al. [25, 34] and Ren et al. [29] have been proposed to improve [22] to create fewer and more natural nearly convex shapes. Both methods [25, 29] use mutex pairs to enforce the concavity constraint. Points  $p_1$  and  $p_2$  form a mutex pair if their straight line connection is not completely inside the given shape. Their focus is on separating all mutex pairs with concavity-based weights larger than a user-specified threshold. Liu et al. [25] used linear programming to compute decomposition with minimum cost. Short cut rule is the only rule considered for evaluating the cost of a cut. Ren et al. [29] applied a dynamic subgradient-based branch-and-bound search strategy to get minimum number of cuts. Other than short cut rule, [29] also took minima rule into consideration which is that a cut resolving at positions with greater negative curvatures is preferred. Similarly, Juengling and Mitchell [17] formulate decomposition of a polygon as an optimization problem and applies dynamic programming to find the optimal subset of cuts from all possible cuts. The objective functions used for optimization favors short cuts that create dihedral angles close to  $\pi$ . Mi and DeCarlo [27] propose to decompose shape into elliptical regions glued by a hyperbolic patches.

An important requirement in shape decomposition is its robustness to boundary noise. Several of these methods require pre-processing (e.g., model simplification [17, 32]) or post-processing (e.g., merging over-partitioned components [11, 27]) due to boundary noise. Other methods [22, 25, 29] are designed to tolerate these noise. However, as far as we know, no existing approaches focused on handling topological noise that appear quite commonly in polygons generated from images with significant noise and overlapping objects.

## 3. Concavity

Concavity and the process of measuring it play the key role in our method (DUDE). To ease the concavity measuring process, we represent 2D shapes using polygons. Let us first define some notations used throughout the paper. A polygon  $P$  is represented by a set of  $n$  boundaries  $\{P_0, P_1, \dots, P_{n-1}\}$ , where  $P_0$  is the external boundary and  $P_{k>0}$  are boundaries of *holes*. Each boundary consists of an ordered set of vertices  $\{p_i\}$  which defines a set of edges. Each edge starts at vertex  $p_i$  as  $e_i = \overline{p_i p_{i+1}}$  and has two

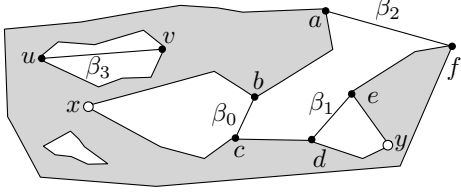


Figure 2. Bridges  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  and a concave feature  $x$ . The end points of  $\beta_3$ ,  $u$  and  $v$ , are the antipodal vertices of the hole. Bridges  $\beta_0$  and  $\beta_1$  are the children of  $\beta_2$ .

associated vectors: the vector  $\vec{v}_i = \overrightarrow{p_i p_{i+1}}$  and the outward normal  $\vec{n}_i$ . Because DUDE performs decomposition in both areas enclosed by  $P$  and  $\overline{P}$ , the complement of  $P$ , it is worth noting that  $\overline{P}$  can be obtained by reversing the ordering of  $\{p_i\}$ . In many cases, polygons are usually extracted from images by detecting the silhouettes, thus usually containing many geometric and topological noises.

Because DUDE relies heavily on the measure of concavity, let us now define the concept of bridge and pocket.

**Definition 1.** A *bridge*  $\beta$  of a given polygon  $P$  is a segment  $\beta = \overline{vu}$  connecting two points  $v$  and  $u$  on the boundary  $\partial P$  of  $P$  from the space exterior to  $P$ . More specifically, a segment  $\overline{vu}$  is a bridge of  $P$  if and only if  $v, u \in \partial P$  and the open set of  $\overline{vu}$  is in the complement  $\overline{P}$  of  $P$ , i.e.  $\overline{vu}^\circ \subset \overline{P}$ .

Therefore, a bridge cannot enter  $P$  or intersect the boundary of  $P$  except at its end points. Examples of bridge are shown in Fig. 2. Note that, unlike bridges defined [22], this definition of bridge can be inside the convex hull of  $P$ .

**Definition 2.** A *pocket*  $\rho$  of a bridge  $\beta = \overline{vu}$  is a subset of the boundary  $\partial P$  connecting  $v$  and  $u$  so that the region enclosed by  $\beta$  and  $\rho$  is completely in  $\overline{P}$ .

Intuitively, when traversing the boundary of  $P$ , a bridge can be viewed as a short cut over its pocket. For example, the pocket of the bridge  $\beta_0$  in Fig. 2 is a polyline between vertices  $b$  and  $c$  via  $x$ .

The relationship between the bridge and pocket gives us an intuitive way to define concavity. For example, in the simplest case, the concavity of vertices in the pocket is simply the straight line distances to the bridge. A complete definition of concavity will be provided in Section 4 for the cases that the pocket is embedded in other pockets.

## 4. Dual Space Decomposition of Polygons

In dual-space decomposition (DUDE), the input polygon  $P$  is decomposed based on the decomposition of the complement  $\overline{P}$  of  $P$  (*dual-space*). Algorithm 1 outlines the idea. The algorithm starts by determining the bridges and pockets from the convex hull  $CH(P)$  of  $P$ . Algorithm 1 then proceeds by decomposing the regions  $\overline{P}_i$  enclosed between the bridge  $\beta_i$  and pocket  $\rho_i$  pair. The cuts generated by these recursively calls  $DUAL-DECOMP(\overline{P}_i, \tau)$  are, by definition,

bridges of  $P$ . Concavity is then measured, important concave features are identified using the bridges from  $CH(P)$  and the cuts for  $\overline{P}_i$ . Finally,  $P$  is decomposed using these concave features. In the rest of this section, we will discuss each of these steps in detail. Figure 3 illustrates  $\overline{P}$  is decomposed to find concave features of  $P$ .

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### Algorithm 1 Dual-Space Decomposition

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- 1: **procedure** DUAL-DECOMP( $P, \tau$ )
  - 2:   Compute convex hull  $CH(P)$  of  $P$
  - 3:   Determine bridges  $\beta = \partial CH(P) \setminus \partial P$
  - 4:   Determine pockets  $\rho$  from  $\beta$
  - 5:   **for all**  $\overline{P}_i$  enclosed by  $\rho_i \in \rho$  and  $\beta_i \in \beta$  **do**
  - 6:      $\kappa = \kappa \cup DUAL-DECOMP(\overline{P}_i, \tau)$
  - 7:   Measure concavity using  $\beta \cup \kappa$    ▷ see Section 4.1
  - 8:    $c = FEATURES(\beta \cup \kappa, \tau)$    ▷ see Section 4.2
  - 9:   **return** CUT( $P, c$ )   ▷ see Section 4.3
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### 4.1. Bridge Hierarchy

The bridges  $\beta$  are obtained from two sources: (1) the convex hull boundary (line 3 in Algorithm 1) and (2) the decomposition of the pockets (line 5 in Algorithm 1). First, we note that these bridges form a hierarchy. More specifically, we say that a bridge  $\beta$  is the parent of  $\beta'$  if the pocket  $\rho$  of  $\beta$  is contained in the pocket  $\rho'$  of  $\beta'$ . For example, in Fig. 2, bridges  $\beta_0$  and  $\beta_1$  are both kids of  $\beta_2$ . It is not difficult to show that this relationship of all bridges can be uniquely determined. That is, when two pockets overlap, one must enclose the other one. As a consequence, we can state the following theorem.

**Theorem 3.** *The hierarchical relationship of bridges determined in Algorithm 1 must form a tree structure.*

As a result, each vertex of  $P$  has a unique path to the convex hull boundary of  $P$ . The length of this path determines the concavity of the vertex.

Note that a hole boundary is a pocket associated with only bridges from the decomposition. When a hole is nearly convex, there is no bridge associated with the hole. Therefore, a pseudo bridge is created by connecting the *antipodal pair*, i.e., two farthest apart vertices, of the hole (see  $\beta_3$  in Figure 2). When a hole is decomposed by DUDE, bridges are formed from the cuts. Given  $k$  bridges, we can form  $k$  bridge hierarchies by using each cut as the root. We choose the hierarchy with lowest depth to measure the concavity of vertices in the hole.

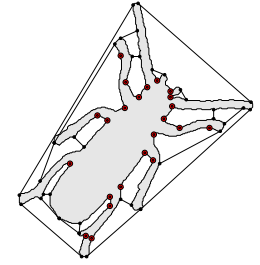


Figure 3. Decomposed  $\overline{P}$ .

Our objective in measuring the concavity is to identify pockets that have large intolerable concavity. Intuitively, the concavity of a pocket is the largest concavity of its vertices.

**Definition 4.** For a pocket  $\rho$  without children, we define:

$$\text{concavity}(\rho) = \max_{v' \in \rho} (\text{dist}(v', \beta)) . \quad (1)$$

Otherwise,  $\text{concavity}(\rho)$  is defined as:

$$\max_{\rho' \in C(\rho)} (\text{concavity}(\rho') + \text{dist}(\beta', \beta)) , \quad (2)$$

where  $C(\rho)$  is the children of  $\rho$ ,  $\beta'$  is the bridge of  $\rho'$ , and  $\text{dist}(x, y)$  is the shortest Euclidean distance between objects  $x'$  and  $y$ .

We say that  $\rho$  an *intolerable pocket* if  $\text{concavity}(\rho)$  is greater than  $\tau$ . Finally, we would like to point out that this hierarchical definition of concavity provides better accuracy and efficiency over the traditional concavity measurement computed directly between vertices and the bridge using either the (fast but inaccurate) straight-line distance or the (accurate but slow) shortest-path distance [22].

## 4.2. Detect Intolerable Concave Features

Given an intolerable pocket  $\rho$  either from the external boundary or a hole boundary of  $P$  and the complement polygon  $\bar{P}$  enclosed by  $\rho$  and its associated bridge, we can determine the intolerable concave features of  $\rho$  by finding the smallest  $CH'$  approximation of the convex hull  $CH(\bar{P})$  of  $\bar{P}$  such that the distance from  $CH'$  to  $CH(\bar{P})$  is smaller than  $\tau$ . Using the idea similar to the variational shape approximation [10], we then obtain an optimal approximation  $CH'_i$  of  $CH(\bar{P})$  with  $i$  vertices in iteration  $i$ . We start from two vertices, i.e.  $i = 2$ , in the approximation and iteratively add a vertex to the approximation to obtain a better approximation until the two-way Hausdorff distance between  $CH'_i$  of  $CH(\bar{P})$  is less than  $\tau$ . The vertices in  $CH'_i$  are the intolerable concave features. An example of the intolerable concave features is shown in Figure 4(a).

Because the complement polygon  $\bar{P}$  of  $\rho$  must be nearly convex (otherwise  $\bar{P}$  is decomposed), it is provable that all the intolerable concave vertices of  $\rho$  must be on the convex hull  $CH(\bar{P})$  of  $\bar{P}$  [14]. In addition, by determining the smallest approximation of  $CH(\bar{P})$  with bounded Hausdorff distance, the method above provides the minimum number of intolerable concave features .

## 4.3. Decompose at Intolerable Concave Features

In this section, we describe how the intolerable concave features of a given polygon  $P$  are connected into *cuts* that segment  $P$  into nearly convex components. Our method starts off by determining the *resolvers* of each concave feature. A resolver of a concave feature  $v$  is a set of diagonals

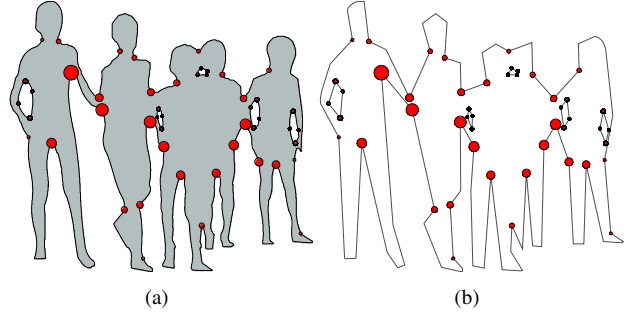


Figure 4. (a) Polygon with identified concave features. Size of the circle indicates the significance of the feature. (b) Simplified polygon using the concave features.

of  $P$  that can locally reduce the concavity of  $v$  to  $\tau$  or less. Once all resolvers from  $P$  are identified and evaluated, our method then proceeds to determine a set of resolvers that maximizes the total scores subject to the constraints that no conflicting resolvers are selected.

In the rest of this section, we will discuss how a resolver is defined and identified in Section 4.3.1. We then address the optimization problem by solving *0-1 integer linear programming* in Section 4.3.2.

### 4.3.1 Resolvers of Intolerable Concave Features

A resolver of an intolerable concave feature  $v$  consists of a set of diagonals incident to  $v$ . We say that a diagonal  $d$  is valid for a given concave feature  $v$  if (1)  $d$  is in the interior of  $P$  and (2) the end points of  $d$  belong to different pockets.

Given a polygon  $P$ , valid diagonals are determined using the Constrained Delaunay Triangulation (CDT) of both  $P$  and the simplified polygon  $\hat{P}$ . The simplified polygon  $\hat{P}$  is composed of intolerable concave features and the vertices between two consecutive bridges. Essentially,  $\hat{P}$  is  $P$  with all pocket vertices replaced by the intolerable concave features. An example of  $\hat{P}$  is shown in Figure 4(b). To find all valid diagonals, we first compute the  $CDT_{\hat{P}}$  of  $\hat{P}$ . Let  $D_{\hat{P}} \subset CDT_{\hat{P}}$  be a set of valid diagonals incident to concave feature of  $P$ . We then compute the  $CDT_P$  using the edges of  $P$  and the diagonals in  $D_{\hat{P}}$  as constraints. Let  $D_P \subset CDT_P$  be a set of valid diagonals incident to concave feature of  $P$ . The valid diagonals  $D_{\hat{P}} \cup D_P$  are candidates for resolvers.

We say that a set of valid diagonals  $D$  can locally resolve a concave feature  $v$  if the *local residual concavities* at  $v$  are all less than  $\tau$ . These local residual concavities at  $v$  due to  $D$  can be computed using only  $D$  and the edges of  $\hat{P}$  incident to  $v$ . Then, a resolver  $\mathcal{R}$  of  $v$  is simply a *minimum set of valid diagonals* that can resolve  $v$ . An example of resolvers for vertices  $v$  and  $x$  is shown in Fig. 5.

The resolvers from a vertex  $v$  are mutually exclusive because a single resolver, by definition, can resolve  $v$ . As a consequence, no two resolvers of  $v$  should be selected in



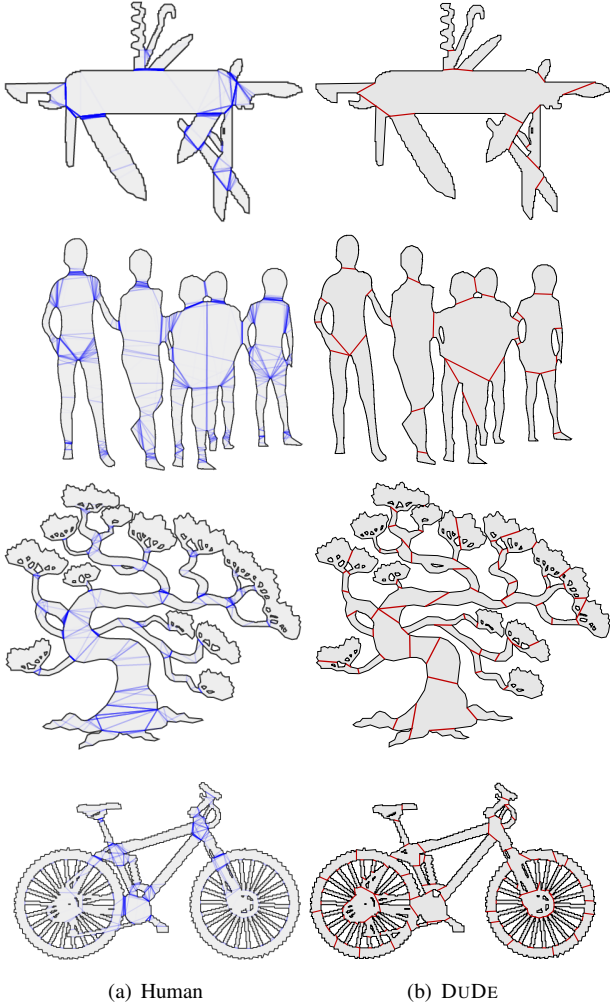


Figure 7. Differences between human segmentation and DUDE.

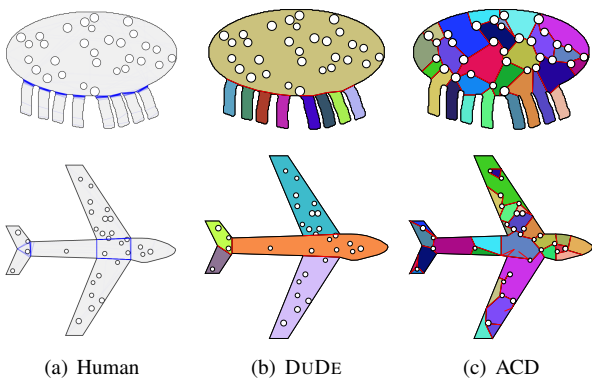


Figure 8. Decomposition results of the aggregated human cuts, DUDE and ACD.

[28] and the other one is called *Cluster Coverage* (CC).

To perform RI evaluation, we first obtained the representative human segments via  $k$ -means clustering for each polygon. Here  $k$  is determined by the mean number of cuts

created by human for a given polygon.

Next, we overlay the decomposition with a  $200 \times 200$  regular grid tightly bounding the polygon  $P$ . A cell is considered as valid if it has more than half of its area inside  $P$ . For a given segmentation, each component in the segmentation is assigned a unique index and the cells enclosed by this component are assigned the same index number. RI evaluation of two segmentations (of the same polygon) is measured by the likelihood that each pair of cells is in the same component of two segmentations [28]. More specifically, let  $S_1$  and  $S_2$  be two segmentations, and  $s_i^1$  and  $s_i^2$  be the indices of cell  $i$  in  $S_1$  and  $S_2$ , respectively. Then their RI is defined as:

$$RI(S_1, S_2) = \binom{2}{N}^{-1} \sum_{i,j,i < j} [C_{ij}P_{ij} + (1 - C_{ij})(1 - P_{ij})],$$

where  $N$  is the number of cells inside the original polygon,  $C_{ij} = 1$  if  $s_i^1 = s_j^1$ ,  $P_{ij} = 1$  if  $s_i^2 = s_j^2$ ,  $C_{ij}P_{ij} = 1$  if cell  $i$  and cell  $j$  have the same index in both  $S_1$  and  $S_2$ , and, finally,  $(1 - C_{ij})(1 - P_{ij}) = 1$  indicates the cell  $i$  and  $j$  have different indices.

A major drawback of the RI-based evaluation is that the size of the representative cuts can affect the RI value dramatically. Therefore, we propose another evaluation method called *Cluster Coverage*. A Cluster Coverage (CC) value between a segmentation  $S$  and clusters  $C$  of segmentation created by the human subjects is determined in the following way:

$$CC(S, C) = \frac{\sum_i^k W_i f_i}{\sum_i^k W_i},$$

where  $W_i$  is the number of elements in  $C_i$  and  $f_i$  is an indicator function that returns one if  $\exists S_j \in C_i$ .

The RI and CC values in Table 1 are obtained from the evaluations between segmentations created by a given method and the representative human segmentations over all the polygons shown in Fig. 6. Therefore, for both RI and CC evaluations, higher score indicates higher similarity to the representative human segmentations. Human vs. Human comparison is used as the baseline for our comparison. From Table 1, we see that the average RI value of DUDE is only slightly below that of Human, while higher than those from MNCD and ACD. This means DUDE provides segmentations closer to the aggregated human segmentations. The average CC value of Human is only slightly below that of DUDE while the standard deviation of DUDE is slightly higher. The CC evaluation again confirms that DUDE provides segmentation closer to the aggregated human segmentation than MNCD and ACD.

### 5.3. Compare with CSD and MNCD

Both CSD [25] and MNCD [29] generate candidate cuts and mutex pairs using Reeb graphs from multiple rotations.

Table 1. Mean values of Rand Index (RI) and Cluster Coverage (CC) and their standard deviations  $SDV_{RI}$  and  $SDV_{CC}$ .

Method	Human	DUDE	MNCD	ACD
RI	0.73	0.68	0.46	0.39
$SDV_{RI}$	0.12	0.21	0.17	0.36
CC	0.67	0.77	0.73	0.40
$SDV_{CC}$	0.094	0.15	0.26	0.38

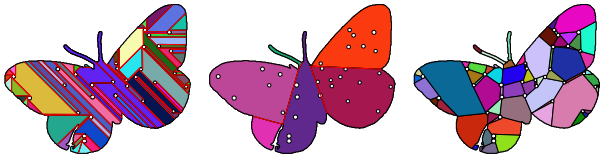


Figure 9. Decomposition result of MNCD and DUDE. From left to right: MNCD ( $\tau = 0.1$ ), DUDE ( $\tau = 0.1$ ), DUDE ( $\tau = 0.01$ ).

The major difference between DUDE and these methods is that DUDE can handle holes (that appear frequently in shapes of many overlapping objects) much more naturally. Figure 9 shows the difference between the decompositions of MNCD and DUDE. Consequently, as we have also seen from Table 1, DUDE can generate more similar results than MNCD compared to human segmentations. The reasons of lower RI and CC scores may also be coming from the fact that there are insufficient optimal candidate cuts in their methods that are generated from intersection of scanning lines and polygon boundary. Moreover, for polygon with complex holes, MNCD requires parameters to balance between the visual naturalness and the number of cuts (while DUDE requires only the value  $\tau$ ). The naturalness may be sensitive to the local noise. Finding good values for these parameters is usually not easy.

## 6. Conclusion and Discussion

In this paper, we proposed a new method: *Dual-space Decomposition* (DUDE) to do 2D shape decomposition, which has applications in region/part-based recognition, skeleton-extraction (Fig. 10), *etc.* DUDE is designed to segment shape composed overlapping objects with a significant number of holes. Using two evaluation methods: Rand Index and Cluster Coverage, we show that segmentation created by DUDE is statistically more similar to the man-made segmentation than those created by other methods.

Since DUDE is developed purely based on the geometric properties, its ability to segment meaningful part is limited. For example, our data shows that people are good at segmenting shapes composed of humans and animals. When a figure contains human shape and other non-human objects, people prefer to separate human from the models.

Although there has been many methods recently proposed in segmenting 3D shapes, 2D part segmentation has its own difficulty because 2D shapes tend to overlap thus

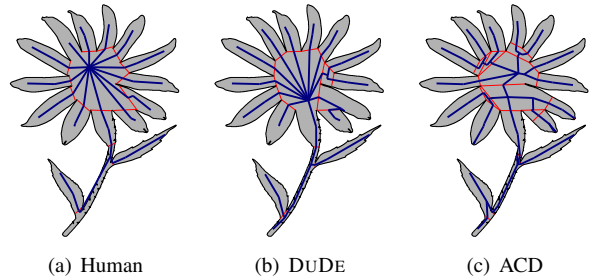


Figure 10. Skeleton extracted from different methods

create ambiguity. When (either human or non-human) shapes overlap, people also tend to segment each individual object or human from the group, even when the cut is long (e.g., the crowd shape in Figure 7). Moreover, different people also assign the overlapping area to different parts. More people tend to assign the overlapping area to an object if it has larger ratio of non-overlapping parts in the whole shape rather than other objects that share this overlapping area. For example, in the bike polygon in Figure 7, when the frame has overlapping area with wheel, more people tend to assign the area to wheel instead of frame.

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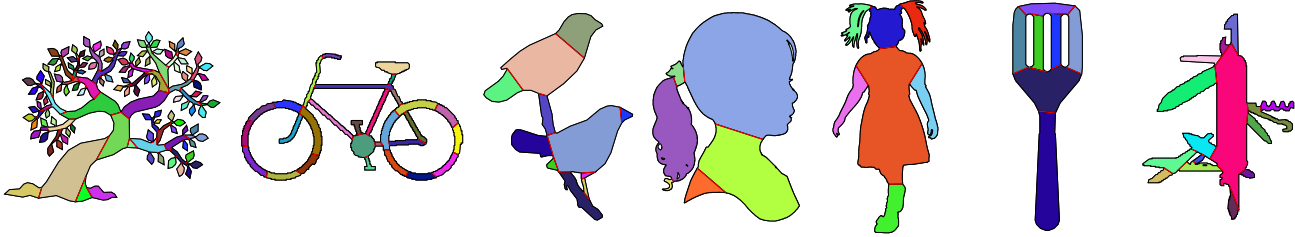


Figure 11. Decomposition results of DUDE for polygons with many holes.

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