Predicting Multiple Structured Visual Interpretations

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Abstract

We present a simple approach for producing a small number of structured visual outputs which have high recall, for a variety of tasks including monocular pose estimation and semantic scene segmentation. Current state-of-the-art approaches learn a single model and modify inference procedures to produce a small number of diverse predictions. We take the alternate route of modifying the learning procedure to directly optimize for good, high recall sequences of structured-output predictors. Our approach introduces no new parameters, naturally learns diverse predictions and is not tied to any specific structured learning or inference procedure. We leverage recent advances in the contextual submodular maximization literature to learn a sequence of predictors and empirically demonstrate the simplicity and performance of our approach on multiple challenging vision tasks including achieving state-of-the-art results on multiple predictions for monocular pose-estimation and image foreground/background segmentation.

1. Introduction

Computer vision tasks such as object recognition [12], semantic segmentation [4], tracking [18], monocular human pose estimation [38] and point cloud classification [27], are often addressed by a pipeline architecture where each module of the pipeline produces several hypotheses as input to the next module. Considering multiple options at each stage is good practice as it avoids premature commitment to a single answer which, if wrong, can jeopardize the quality of decisions made downstream [10, 37]. As an example consider Figure 1 where multiple predictions are generated for a foreground/background segmentation task. We see that the prediction with the highest confidence (denoted by prediction 1) can be far from the groundtruth. The principal requirement of a list is that at least one hypothesis in the list is close to the groundtruth labeling (high list recall). A characteristic of lists which achieve high recall in a small number of hypotheses is diversity [32] which increases the odds of at least one accurate prediction.

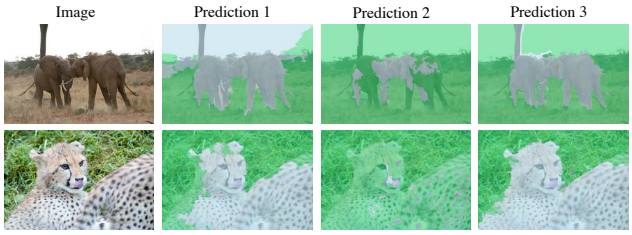


Figure 1: For a given image our method trains a small number of structured predictors in sequence. For a test image, the list of predictors are invoked to produce multiple hypotheses. Our approach produces high recall lists within a small number of hypotheses and can use any structured predictor available.

Such fixed length lists with high recall need to be considered in many other non-vision tasks including information retrieval, server job scheduling, document summarization, grasping and motion planning. For example in information retrieval, when a user searches for the term "jaguar" on a search engine, the user may either intend to search for the animal, the car, or the football team. Search results must include highly ranked links to all three possible interpretations [32]. These problems have received much attention in recent literature [7, 8, 23, 24, 32, 33, 34, 40, 41].

Our central insight is that diversity in a list of structured predictions need not be enforced, but that it is an emergent property of optimizing the correct submodular recall objective. Our procedure trains a sequence of predictors, each of which produces a hypothesis, which constitutes the list.

Submodular functions are characterized by a diminishing returns property that implies that any list that maximizes a monotone submodular function will implicitly produce diverse hypotheses. This is due to the fact that adding hypotheses which are similar to previously chosen hypotheses in the list results in small marginal gains. For example, in the semantic scene labeling problem it is not beneficial to predict a labeling in the second position of the list which differs from the first labeling in the list only by a few pixels. Note that the desired property is to achieve high recall, while diversity is merely a characteristic of lists that achieve

this. Therefore our objective optimizes for recall and does not explicitly enforce diversity, instead the maximization of the submodular objective naturally produces diverse hypotheses. Conveniently, submodular monotone functions can be maximized efficiently by greedily maximizing the marginal benefit which ensures performance within $1 - \frac{1}{e}$ ($\sim 63\%$) of the optimal list of items of a fixed length [28]. Since monotone submodular functions exhibit this diminishing returns property as the list of input arguments grows, they have been harnessed to serve as convenient objective functions for optimizing both low-error items and diversity [7, 24, 32, 40].

The vision tasks mentioned above are generally posed as structured prediction problems. Making a single best prediction in structured problems is difficult due to the combinatorially large state space that has to be considered. While a number of approaches, both probabilistic [19, 22] and margin-based [35, 36], for learning and inference in structured problems are well known, methods for making multiple predictions in structured problem domains are relatively few [2, 14, 20, 30]. We develop a learning-based approach to produce a small list of structured predictions that ensures high recall in a variety of computer vision tasks.

In contrast to recent developments which train a single model during the learning phase and modify the inference procedure to produce multiple hypotheses at test time [2, 20, 30], our approach trains separate predictors during the learning phase to produce each of the hypotheses in the list. This alternate approach has several advantages—the learning procedure is optimized for the task of producing a list with high recall; diversity does not need to be enforced in an ad hoc fashion but is an emergent property of lists that maximize our objective; it is agnostic to the inference method used and can be utilized for any class of structured predictor. The primary contributions of our approach are:

- Our approach is a *model agnostic* framework applicable to extending any structured prediction algorithm to make multiple predictions. In any task domain for which learning algorithms exist to generate a single best prediction, our approach can be employed for making multiple predictions by training multiple instances.
- Our approach is *parameter free*. In contrast, current state-of-the-art approaches enforce diversity by explicitly introducing a diversity modeling term in the objective function. Such parameters are tuned on validation data. It is not clear that artificially enforcing diversity in such a way is the right thing for the task at hand to achieve the best performance [5, 26].
- We study the empirical performance of our approach and demonstrate state-of-the-art results on multiple predictions for monocular pose estimation and foreground/background segmentation on benchmark datasets.

2. Related Work

Significant prior work exists in the non-structured output domain. Dey et al. [7] leveraged the work of Streeter and Golovin [34] and proposed a reduction of list prediction to training a list of classifiers or regressors (Contextual Sequence Optimization – ConSeqOpt). This allowed the approximate greedy procedure proposed by Streeter and Golovin [34] (independently discovered by Radlinski et al. [32]) to be used with features of the task environment. Ross et al. [33] extended [34] and ConSeqOpt to predict lists by training a single predictor instead of a list of predictors. This procedure makes the training step more data efficient. Kulesza et al. [21] have adapted determinantal point processes (DPP), a model used in particle physics for optimizing for diverse but low error predictions. DPPs are especially attractive because they allow for efficient, exact inference procedures and are similar to submodular optimization methods.

The related work in multiple structured prediction can be grouped into two categories: 1) The first are methods which are *model-dependent*. These methods are tied to the specific learning and inference procedure being used (e.g. S-SVM, CRF) and cannot easily be adapted to different structured prediction methods. 2) The second category of models are *model-agnostic*, which are not tied to the specifics of the chosen structured prediction method.

Model-dependent methods: Batra et al. [2] deal with the problem of inferring low error and diverse solutions from a Markov Random Field (MRF). They approach this problem by introducing a constraint in the MRF objective function which says that a new solution must be at least some distance away from each of the previous solutions. The constraint is moved to the objective by a Lagrangian multiplier λ and then solved using a supergradient algorithm. λ is treated as a free parameter and is optimized over a validation set. They term this approach as DivMBest. Note that a single model is initially learnt (the MRF) and only during inference time diverse solutions are obtained by imposing constraints on the inference procedure. Nilson et al. [29] and Weiss et al. [39] propose methods for using loopy belief propagation for finding the most probably solutions at inference time in graphical models. But their methods don't try to incorporate diversity to improve performance. Park and Ramanan [30] use a modified version of standard max-product inference which aims to enforce diversity by incorporating part-overlap constraints, which they term as NBest.

In the approach proposed by Guzman-Rivera et al. [14], a structured SVM [36] (S-SVM) is trained for each position of the list. During inference time, each S-SVM is invoked to predict a structured output. They minimize an upper bound of the non-convex, structured hinge loss via a kmeans-based initialization step and an expectation-maximization (EM) style coordinate-descent minimization

algorithm. They term their approach as Multiple Choice Learning (MCL). In more recent work, Guzman-Rivera et al. [15] explicitly add diversity to the MCL objective and optimize a surrogate via an EM style block coordinate-descent minimization routine similar to MCL. An extra parameter which trades off between diversity and accuracy is then tuned via cross-validation. They term this approach as Diverse Multiple Choice Learning (DivMCL).

In comparison to such model-dependent methods, our proposed method is model-agnostic and can use *any* structured prediction approach.

Model-agnostic methods: To the best of our knowledge, the only such method is the ad hoc boosting-like weighting scheme used in [14, 16] which we denote henceforth as GR14. The weighting scheme of GR14 [16] has been used for specific task of camera re-localization. This method has a free parameter which must be tuned on validation data. In comparison our approach is parameter free and achieves comparable or better results on standard vision tasks.

3. Background: Submodular Functions

The operator $\oplus$ denotes order dependent concatenation of lists and $\mathcal{V}$ denotes the space of all possible items which can be used for constructing lists. A function which takes a list of items as input $f : \mathcal{S} \rightarrow [0, 1]$ is monotone, submodular for any sequence $S = [\![a_1, a_2 \dots a_{|S|}]\!] \in \mathcal{S}$ where $\mathcal{S}$ is the space of all possible lists of items $a_{1\dots|S|} \in \mathcal{V}$, if it satisfies two properties. 1). *Monotonicity*: for any list $S_1, S_2 \in \mathcal{S}$, $f(S_1) \leq f(S_1 \oplus S_2)$ and $f(S_2) \leq f(S_1 \oplus S_2)$ 2). *Submodularity*: for any list $S_1, S_2 \in \mathcal{S}$, and any particular item $a \in \mathcal{V}$, $f(S_1 \oplus S_2 \oplus [\![a]\!]) - f(S_1 \oplus S_2) \leq f(S_1 \oplus [\![a]\!]) - f(S_1)$. A simple way of maximizing such a monotone, submodular function f is by greedily choosing an element a which maximizes the benefit of adding it to the pre-existing list S, i.e., $a^* = \arg\max_{a \in \mathcal{A}} \; (f(S \oplus [\![a]\!]) - f(S))$ where a^* is the next element to be added to the list. Nemhauser et al. [28] proved that for an input list of fixed length, the greedy algorithm achieves at least $1 - \frac{1}{e}$ of that achieved by the optimum list of the same length.

4. Predicting Multiple Visual Interpretations

Structured prediction problems in machine learning and computer vision are characterized by a multidimensional *structured* output space $\mathcal{Y}$, where the notion of *structure* varies according to the problem. For example, in semantic scene understanding, the structure in the output $\mathbf{y} \in \mathcal{Y}$ refers to the fact that nearby regions in the image tend to have correlated semantic labels. In human pose estimation from images, the location of a limb in the image is correlated with the locations of other limbs.

One possible approach to structured predictions could be to use the well understood approach of multi-class classification by treating each possible structured output as a label. If this were possible, multiple low error and diverse interpretations could be directly generated using a scheme such as ConSeqOpt [7]. However, the challenge in such structured prediction tasks is that the space of possible output variable combinations is exponential in the number of labels for each variable. For example for an image with 10^4 pixels and 21 possible labels for each pixel there are $(21)^{(10^4)}$ possible labelings. This is also why structured prediction tasks cannot be addressed by multi-class classification as the number of classes is exponentially large. As a result, directly applying a procedure such as ConSeqOpt to generate multiple interpretations is infeasible.

Our approach is inspired by the ideas set forth in [7, 8, 33]. We define a monotone submodular function over a sequence of structured predictors and show that a simple greedy algorithm can be used to train a sequence of such predictors to produce a set of structured predictions with high recall. More formally our problem can be stated as follows.

Problem Statement: The goal of our approach is, given an input image $I \in \mathcal{I}$, to produce a sequence of N structured outputs $Y(I) = [\![\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_N]\!] \in \mathcal{Y}$ with low error and high recall. We formulate this as the problem of learning a sequence of structured predictors $S = [\![\pi_1, \pi_2, \dots, \pi_N]\!]$ where each predictor $\pi_i : \mathcal{I} \rightarrow \mathcal{Y}, \pi_i \in \Pi$, in the sequence produces the corresponding structured output $\mathbf{y}_i$, where Π is a hypothesis class of structured predictors and $\mathcal{Y}$ is the space of structured predictions.

We begin by describing a submodular objective that captures the notion of low error and high recall. Let us denote j^{th} training sample as a tuple $\{(I^j, \mathbf{y}^j_{gt})\}_{j \in 1\dots|\mathcal{D}|}$, where for each image I^j, the ground truth structured label is denoted by $\mathbf{y}^j_{gt}$. We denote by $l : \mathcal{Y} \times \mathcal{Y} \rightarrow \mathbb{R} \in [0, 1]$, a *loss* function that measures the disagreement between the predicted structured output $\mathbf{y}$ and the ground truth structured label $\mathbf{y}_{gt}$. The corresponding function measuring *gain* is thus given by $\mathbf{g}(\mathbf{y}, \mathbf{y}_{gt}) = 1 - l(\mathbf{y}, \mathbf{y}_{gt})$. We define a sequence of structured outputs as:

$$Y_S(I) = [\![\pi_i(I)]\!]_{i \in 1\dots N} \tag{1}$$

We then define the quality function,

$$f(Y_S(I), \mathbf{y}_{gt}) = \max_{i \in 1,\dots,N} \{\mathbf{g}(\pi_i(I), \mathbf{y}_{gt})\}, \tag{2}$$

$$= 1 - \min_{i \in 1,\dots,N} \{l(\pi_i(I), \mathbf{y}_{gt})\} \tag{3}$$

that scores the sequence of structured predictions $Y_S(I)$ by the score of the best prediction produced by the sequence of predictors $S = [\![\pi_1, \pi_2 \dots, \pi_N]\!]$. We note that to maximize this scoring function with respect to the sequence of predictions at least one of the predictions $\mathbf{y}_i$ in the sequence needs to be close to the ground truth. In order to learn a sequence of predictors that works well across a distribution of the data, the objective function we would like to optimize is

Algorithm 1 `train_seqnbest`

Input: Sequence length N, structured prediction routine $\pi \in \Pi$, dataset $\mathcal{D}$.
$S = \{\}$, $\{\mathbf{w}_0^j = 1\}_{j \in 1 \ldots |\tilde{\mathcal{D}}|}$
for $i = 1$ **to** N **do**
  $\pi_i = \texttt{train\_struct\_predictor}(\tilde{\mathcal{D}}, \mathbf{w}_{i-1})$
  $S \leftarrow S \oplus \pi_i$
  $\mathbf{w}_i = \texttt{compute\_marginal\_weights}(S, \tilde{\mathcal{D}})$
end for
Return: $S = [\![\pi_1, \pi_2, \ldots, \pi_N]\!]$

the expected value of the above function over the distribution of the data $\mathcal{D}$:

$$F(S, \mathcal{D}) = \mathbb{E}_{(I, \mathbf{y}_{gt}) \sim \mathcal{D}} \left[f(Y_S(I), \mathbf{y}_{gt}) \right]. \tag{4}$$

The resulting optimization problem is therefore to find the sequence of predictors S that maximizes the objective function F in Equation 4 and can be written as follows:

$$\max_S \mathbb{E}_{(I, \mathbf{y}_{gt}) \sim \mathcal{D}} \left[f(Y_S(I), \mathbf{y}_{gt}) \right]. \tag{5}$$

The function F of the form in Equation 4 can be shown to be a monotone submodular function over lists of input items as shown in [8] and reproduced in supplementary material for convenience. The natural approach for submodular optimization problems of the form in 5 is to use a greedy algorithm [28]. In each greedy step i, we add the structured predictor π_i^*, that maximizes the marginal benefit as in Section 3. For our objective, maximizing the marginal benefit is written as:

$$\pi_i^* = \arg\max_{\pi \in \Pi} F(S_{i-1} \oplus [\![\pi]\!], \mathcal{D}) - F(S_{i-1}, \mathcal{D}). \tag{6}$$

Maximizing the marginal benefit, as written above, over the space of structured predictors by enumeration is difficult, because there can be uncountably many such predictors. *Instead, we take the approach of directly training a structured predictor to maximize the marginal benefit.* As we do not have access to the true distribution of the data, we maximize the marginal benefit using the empirical distribution $\tilde{\mathcal{D}}$. We denote the loss $l_i^j = l(\mathbf{y}_i^j, \mathbf{y}_{gt}^j)$ as shorthand for the loss of the i^{th} predictor on the j^{th} training sample. Rewriting the objective with respect to the empirical data distribution and in terms of the loss per example we have,

$$\begin{aligned} & F(S_{i-1} \oplus \pi, \tilde{\mathcal{D}}) - F(S_{i-1}, \tilde{\mathcal{D}})) \\ &= \sum_{j \in \tilde{\mathcal{D}}} \left(\min\, [\![l_1^j, \ldots, l_{i-1}^j]\!] - \min [\![l_1^j, \ldots, l_i^j]\!] \right), && (7) \\ &= \sum_{j \in \tilde{\mathcal{D}}} \max \left(\min\, [\![l_1^j, \ldots, l_{i-1}^j]\!] - l_i^j,\ 0 \right), && (8) \\ &= \sum_{j \in \tilde{\mathcal{D}}} \max \left(\xi_{i-1}^j - l_i^j,\ 0 \right). && (9) \end{aligned}$$

Algorithm 2 `compute_marginal_weights` $(S, \mathcal{D})$

Input: Current sequence of trained structured predictors S, Dataset $\tilde{\mathcal{D}}$
for $j = 1$ **to** $|\tilde{\mathcal{D}}|$ **do**
  $l = \{\}$
  for $i = 1$ **to** $|S|$ **do**
    $l_i = l(\pi_i(I^j), \mathbf{y}_{gt}^j)$
    $l \leftarrow l \oplus l_i$
  end for
  $\xi \leftarrow \min(l)$
  if SeqNBest1 **then**
    $w^j \leftarrow \xi$
  else if SeqNBest2 **then**
    $w^j \leftarrow \xi^3 / \left(3\xi^2 - 3\xi + 1\right)$
  end if
end for
Return: $\mathbf{w} = \{w^1, w^2, \ldots, w^{|\tilde{\mathcal{D}}|}\}$

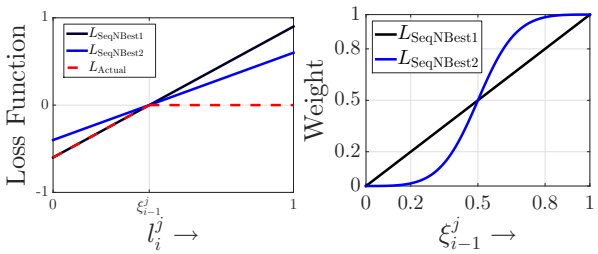
where ξ_{i-1}^j in 9 is the minimum loss obtained by the sequence of $i-1$ predictors on j^{th} sample till now. Since training procedures for structured predictors are usually implemented to minimize loss we rewrite 9 and 6 as:

$$\begin{aligned} \pi_i^* &= \arg\max_{\pi \in \Pi} \sum_{j \in \tilde{\mathcal{D}}} \max \left(\xi_{i-1}^j - l_i^j,\ 0 \right) && (10) \\ &= \arg\min_{\pi \in \Pi} \sum_{j \in \tilde{\mathcal{D}}} \min \left(l_i^j - \xi_{i-1}^j,\ 0 \right) && (11) \end{aligned}$$

Let us denote the per-example desired loss $l_{\text{Actual}} = \min \left(l_i^j - \xi_{i-1}^j,\ 0 \right)$ which is the summand in Equation 10. Consider the relationship of the loss l_{Actual} as a function of the loss of the current predictor l_i^j and the best loss seen before the current predictor (ξ_{i-1}^j). This is drawn in Figure 2a and denoted by the line l_{Actual}. We observe that if a predictor obtains a loss greater than the previous best, ξ_{i-1}^j on an example it does not contribute towards lowering of the loss defined in Equation 10. Whereas if it achieves loss less than ξ_{i-1}^j, it lowers the objective by the same amount that it is less than ξ_{i-1}^j. Optimizing such a loss directly tends to be difficult as it can require modifications that are specific to the structured predictor's training procedure. Instead, we take the approach of optimizing a tight linear upper bound of the loss ($l_{\text{SeqNBest1}}$ in Figure 2a) which results in a procedure that only requires re-weighting the training data and is model-agnostic. Consider a linear upper bound on l_{Actual} defined by the parameter w_i^j,

$$w_i^j l_i^j \geq l_{\text{Actual}}. \tag{12}$$

Training a predictor which optimizes the surrogate loss on the left hand side of 12 is equivalent to training a structured predictor which weights each data sample with the weight w_i^j:



(a) (b)

Figure 2: (a) Upper bound relationship between the proposed surrogate loss and the desired actual loss function (b) Comparison between the proposed weighting schemes as a function of the best-loss seen so far.

$$\sum_{j \in \tilde{\mathcal{D}}} w_i^j l_i^j \geq \sum_{j \in \tilde{\mathcal{D}}} \min\left(l_i^j - \xi_{i-1}^j,\ 0\right) \tag{13}$$

Note that by setting the weight of each sample to be proportional to the marginal benefit left ($w_i^j \propto \xi_{i-1}^j$) we are *minimizing a tight linear upper bound* of the actual loss function we wish to minimize (l_{Actual}). This relationship is indicated by the line $l_{\text{SeqNBest1}}$ in Figure 2b. Our training procedure might be reminiscent of boosting [11] where several predictors are combined to produce a single output. In contrast, our procedure is trained to produce a sequence of predictors each of which makes a separate prediction in a sequence of predictions. Additionally we are also optimizing a completely unrelated loss function.

An alternative tight linear upper bound can be calculated by minimizing the L_2 norm between l_{Actual} and a linear loss function given by $w_i^j l_i^j$. Consider a family of linear upper bounds of the quality function l_{Actual} which has the form

$$l_{\text{SeqNBest2}} = w_i^j l_i^j + b. \tag{14}$$

Minimizing the L_2 distance between l_{Actual} and $l_{\text{SeqNBest2}}$ (See supplementary material for the derivation details) we obtain the following weighting scheme:

$$w_i^j = \frac{(\xi_{i-1}^j)^3}{3(\xi_{i-1}^j)^2 - 3\xi_{i-1}^j + 1}. \tag{15}$$

The graphical relationship between ξ_{i-1}^j and the optimal weight (slope of the line) in $l_{\text{SeqNBest2}}$ is shown in Figure 2a. We see that $l_{\text{SeqNBest1}}$ weights the examples directly proportional to the previous best loss ξ_{i-1}^j, while $l_{\text{SeqNBest2}}$ tends to aggressively upweight hard samples which have high best previous loss ($\xi_{i-1}^j > 0.5$) and aggressively downweights easier examples which have low best previous loss ($\xi_{i-1}^j < 0.5$).

We summarize our algorithm in Algorithm 1. We begin by assigning a weight of 1 to each training sample in the dataset. In each iteration i, we train the structured predictor

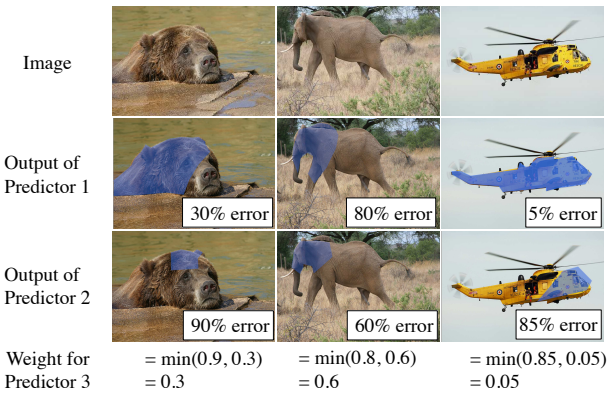


Figure 3: Illustration of SEQNBEST training procedure. Consider a toy training dataset of 3 images (chosen from the iCoseg dataset [1], where the task is to do foreground/background separation. The first predictor gets 30% pixel error on the bear image, while the second predictor gets 90% pixel error. Intuitively, since the first predictor did well already on this image, we should not try as hard on this image compared to the elephant image where none of the 2 predictors did very well. The rule for weighting data points for training the next predictor is minimum of the error by the previous predictors and the last column shows this being applied to this contrived example. Note that the elephant image has the highest weight since none of the previous predictors did well on it, while the helicopter one has the lowest weight, since the first predictor did really well on it.

π_i with the dataset $\mathcal{D}$ and associated weights for each training sample $\mathbf{w}$ and append it to the sequence of predictors S. We recompute the weights $\mathbf{w}$ using the scheme described in Algorithm 2. We iterate for the specified N iterations and return the sequence $S = [\![\pi_1, \ldots, \pi_N]\!]$ of structured predictors. We term this simple but powerful approach as "Sequential N-Best" or SEQNBEST.

4.1. An Example

As an example, on the task of image segmentation the required inputs are the number of predictions we want to make per example N, training dataset $\mathcal{D}$ and the structured prediction procedure for learning and inference π. We explain the algorithm via a toy dataset of 3 images (See Figure 3), where the task is to perform foreground/background segmentation by marking each pixel with either the foreground or background label. Assume that we have trained 2 predictors already, and are calculating the importance (weight) of each image for the 3rd predictor. The second and third columns show the performance of the two predictors on these images. Note that none of the predictors do well on the image of the elephant, however one of the predictors does really well on the helicopter. This tells us intuitively, that training of the third predictor should concentrate more on the image of the elephant, but not as much on the other two since at least one of the previous predictors have done relatively well on it. The last column in the figure shows

the weights for each image which is the minimum of the errors obtained by all previous predictors. This weighting rule achieves the desired behavior of working harder on examples which none of the previous predictors have performed well on.

5. Experiments

We evaluate our methods against both *model-dependent* [2, 14, 30] and model-independent methods [16] (See Section 2). Note that the weighting scheme of GR14 [16] has been used for the specific task of camera re-localization and published results on standardized datasets do not exist. We make a best effort comparison by reimplementing their method for standardized tasks.

We demonstrate that using our simple yet powerful weighting scheme results in better performance than model-dependent methods and comparable or better performance for model-agnostic methods with much less computation due to lack of parameter tuning step.

5.1. Case Study: Human Pose Tracking in Monocular Sequences

In monocular pose estimation the task is to estimate the 2D locations of anatomical landmarks from an image. The task is challenging due to the large variation in appearance and configuration of humans in images. Additional challenges are posed by partial occlusions, self-occlusions, and foreshortening. A related task is to track the pose of a human subject through a sequence of frames of video. In the tracking by detection paradigm of human pose tracking, multiple hypothesis poses are generated per frame of video and then stitched together using a data association algorithm. This avoids making hard commitment to a single best pose at a frame. As long as the correct pose is present amongst the multiple hypothesized poses for each frame, the algorithm can have a chance at picking the correct one using additional temporal information.

Datasets: We evaluate our method on producing multiple predictions for each image in the PARSE dataset used introduced by Yang and Ramanan [38] and on the tracking datasets introduced in Park and Ramanan [30] named "lola", "lola", "walkstraight" and "baseball". We use the same model, code and training set as Yang and Ramanan [38] and use our two weighting methods to train N models as detailed in Algorithm 1 to produce 4 models. We use the same test set used by Yang and Ramanan to compare the average percentage of correct parts (PCP) of the *best* pose as the number of pose hypotheses is increased from 1 to 4.

Analysis Figure 4 shows that as the number of hypotheses is increased SEQNBEST1 and SEQNBEST2 find accurate poses earlier in the sequence than NBEST. The figure plots the average across the test set, of the best pose predicted as the number of pose hypotheses is increased. Batra et al.[2] refer to this as the "oracle" accuracy of a sequence of

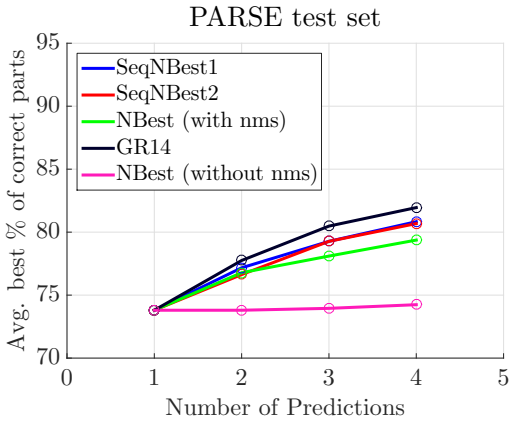


Figure 4: As the number of pose hypotheses allowed is increased from one to four, SEQNBEST predicts more accurate poses compared to NBEST with non-maximum suppression. Both start out at 73.8% percentage of correct parts since the first position's model is identical to both but by the 4^{th} position SEQNBEST has achieved 81.61% average best accuracy while NBEST achieves 79.37%.

predictions. We show results with NBEST with and without non-maximum suppression post processing. Note that even with non-maximum suppression, NBEST is unable to outperform SEQNBEST, which requires no post-processing step. We also compare against the boosting-like weighting scheme of GR14 [16]. GR14 performs marginally better than SEQNBEST, achieving 81.95% oracle accuracy compared to SEQNBEST1's 80.83% by position 4. Note that this boosting-like weighting scheme has a free parameter which is tuned by cross-validation, while we are parameter free. We used the exact same set of four values of this free parameter as used in [16] to tune it for all our following experiments. It takes $\sim$ 24 hours to train a single model by [38] on training data. So GR14 takes 3 times as long to train using cross-validation as SEQNBEST or equivalently 3 times the computational cost accounting for parallelization.

In Table 1 we compare the performance of DIVMBEST with respect to SEQNBEST1 and SEQNBEST2. Three models were trained using the two SEQNBEST schemes on the PARSE training set and then compared to the "oracle" PCPs reported by NBEST and DIVMBEST. In each video sequence SEQNBEST1 or SEQNBEST2 achieves higher recall. In the "walkstraight" dataset SEQNBEST1 achieves 98.5% PCP in 3 positions where DIVMBEST needs 100 predictions to reach the same accuracy. Similarly for "lola1", 20 predictions, "lola2", 7 predictions and for "baseball" 7 predictions are needed by DIVMBEST to reach the same "oracle" PCP as SEQNBEST2. Note that GR14 after much tuning on validation data is still behind SEQNBEST on all four videos.

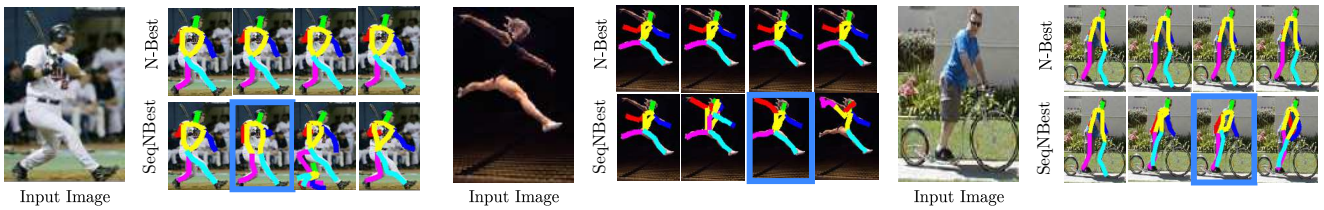


Figure 5: For each of the three images the top row is the sequence of 4 pose hypothesis by NBEST while the bottom 4 are by SEQNBEST. For baseball player SEQNBEST predicts the correct pose in the 2^{nd} guess, for the gymnast in the 3^{rd} guess and the 4^{th} guess for the cyclist. Note that in each case SEQNBEST1 produces poses which are diverse from each other while trying to be relevant to the scene. In each case NBEST produces poses which are almost identical to each other and none of which are close to the ground truth pose.

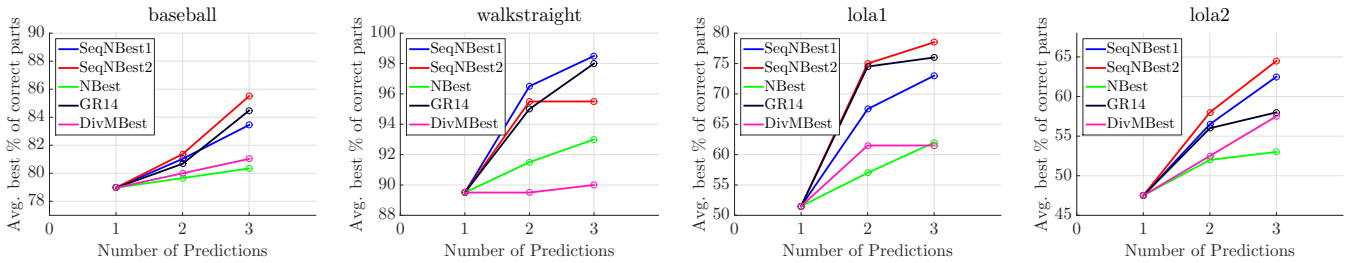


Table 1: Comparison of SEQNBEST1 and SEQNBEST2 to NBEST and DIVMBEST. The average best PCP plotted as the budget for generating hypotheses is increased. In each case SEQNBEST1 and/or SEQNBEST2 predicts more accurate poses for the number of hypotheses allowed.

5.2. Case Study: Image foreground separation

We apply our method to the task of foreground/background segmentation where the task is to assign each pixel in an image with either the foreground or background label.

Dataset: We use the set of 166 images of the iCoseg dataset [1], spanning 9 different events, as used by MCL [14]. The dataset is roughly, equally split into training, validation and test sets. The exact splits were provided to us by the authors of MCL.

Analysis: We compare the performance of SEQNBEST to MCL in two ways: 1) We use the exact implementation of S-SVM provided to us by the authors of MCL as the structured predictor routine in SEQNBEST to train 6 predictors 2) Secondly, *to showcase the flexibility* of SEQNBEST to use *any* structured predictor available, we use the Hierarchical Inference Machine (HIM) algorithm by Munoz et al. [27] to train SEQNBEST. We use texture and C-SIFT [13] as features. Figure 6 (left) shows the "oracle" accuracy of a list of predictions. Additionally we compared against GR14 [16]. We find that using the *same predictor and features* as in MCL, SEQNBEST1 and MCL have comparable performance in Figure 6 (left). When HIM is used as the structured predictor (Figure 6 (right)), it performs much better from the first position and obtains 6% average best error in 6 predictions. The reduction of error stops after the first 3 positions because the HIM model starts approaching the

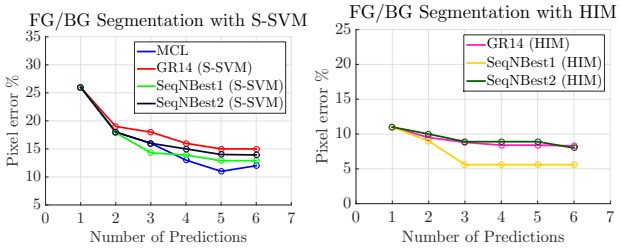


Figure 6: Average best pixel error in the image background, foreground segmentation task as number of predictions are increased. SEQNBEST (with S-SVM) uses the same S-SVM structured predictor routine as MCL.

theoretical limits of its performance on the test set, which is 2% (this was obtained by training and testing HIM on the test set itself).

In summary, variants of SEQNBEST performed on par with model-dependent methods like MCL, which have the advantage of leveraging the specifics of the chosen structured predictor (in this case S-SVM). SEQNBEST, however, is *model-agnostic* and can be readily applied to any structured predictor. We find that SEQNBEST used in conjunction with HIM outperforms the other model-agnostic method, GR14, which is also trained with HIM as the base predictor (Figure 6 (right)). This also serves as an example of SEQNBEST's flexibility in being able to plug-in any powerful predictor.

Table 2: As the number of predictions is increased, we observe a 10.60% gain in "oracle" accuracy over a single prediction on the PASCAL VOC 2012 `val` dataset.

Position	1	2	3	4	5
Oracle acc. (%)	42.91	45.96	46.44	47.09	47.46

5.3. Case Study: Image Segmentation

As mentioned earlier semantic scene segmentation is a very challenging task, where every pixel in an image has to be assigned a semantic label like "boat", "sky" etc. In this section we show initial promising results with SEQNBEST. Note that these are not meant to be competitive with the most recent state-of-the-art advances in image segmentation but meant to showcase the flexibility of our approach in using *any* predictor.

Dataset: In PASCAL VOC 2012 segmentation challenge [9] the task is to mark every test image with one of 20 class labels or the `background` class. Figure 7 shows some example images and their annotated groundtruth labels. There are 1464 images in `train` and 1449 in the `val` set which we use as the test set in our experiments below.

Analysis: We use the Hierarchical Inference Machine (HIM) algorithm by Munoz et al. [27] to learn 5 structured predictors in the SEQNBEST framework. We use the output of category-specific regressors of [3] as additional features to HIM. In the first position HIM achieves 42.91% average intersection/union accuracy over all 21 classes. Table 2 shows the "oracle" accuracy as the number of predictions is increased to 5 where the "oracle" accuracy is 47.46% which is a 10.6% gain.

Prasad et al. [31], have proposed inference procedures for extracting diverse hypotheses in MRFs using various higher-order potentials [6]. This is another example of the *model-dependent* category of methods as described in Section 2. Similar to us, they have demonstrated their method on the semantic segmentation challenge in PASCAL VOC 2012 `val` set. They show impressive "oracle" gains of $\backsim$ 12% over a single prediction. Since their model and code is not yet available, it is not currently possible to directly compare against SEQNBEST. We use a *different* model to achieve similar boosts. Again, this showcases the ease of use and generality of our approach. Note that we are not constrained to specific models or specific diversity terms which may be only compatible with particular model representations.

In ongoing experiments we are using recent advances in convolutional neural networks [17, 25] as the structured predictor for generating multiple segmentations using SEQNBEST.

Figure 7: Qualitative examples of multiple semantic scene segmentations on the PASCAL VOC 2012 dataset. Each predictor tries to get right what the previous predictors have not been able to cover well. For example the cow grazing scene the first two predictors miss parts of the cow while the third one gets majority of it correct.

6. Conclusion

We developed and experimentally validated a simple method for making a high quality *sequence* of structured predictions by learning multiple predictors for each slot of the sequence. The technique presented here directly targets the problem of interest (a good list of predictions) and is easily applied in a black box fashion to a very broad set of structured learners. Moreover, we show it is easily implemented and effective. In contrast to previous methods, we train multiple models during learning as opposed to modifying the inference procedure at test time. While this informs the learning process of the eventual task of producing multiple outputs, it can potentially be inefficient when training each model is expensive. Future work will consider the trade-offs and potential combination of learning multiple models and performing multiple rounds of inference for the problem of producing a good sequence of predictions. Additionally, our method currently trains the predictor in each slot by providing the learning procedure with an upper bound on the marginal benefit for each example. An interesting direction for future work would be to explore methods for accurately communicating the loss of each possible labeling during training at each stage in the sequence in a model-agnostic fashion.

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