Online Sketching Hashing

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Recently, hashing based approximate nearest neighbor (ANN) search has attracted much attention. Extensive new algorithms have been developed and successfully applied to different applications. However, two critical problems are rarely mentioned. First, in many real-world applications, data becomes available continuously in streaming fashion. However, most of the existing hashing models are based on a batch learning fashion. That is to say, when new data arrives, they have to accumulate all the available data and re-train new hash functions, which is apparently a less efficient learning manner for streaming data. Second, for truly large scale datasets, data is usually stored on a distributed disk group and is too large to be read into memory. Moreover, one processor is often incapable of handling the large scale datasets in a feasible amount of time.

In this paper, we propose a novel online hashing approach to address the two problems mentioned above simultaneously. The proposed method is largely inspired by the idea of data sketching [4, 5]. We assume that the data is available in a stream form. Let \( D_t \) denote the data chunk received at round \( t \), where \( t = \{1, 2, \cdots, \} \). In particular, \( D_t = [x_1, x_2, \cdots, x_m] \in \mathbb{R}^{d \times m} \) contains \( m_t \) samples. The mean of the data chunk \( D_t \) is denoted by \( \mu_t \). \( X_t \) denotes the data matrix accumulated from round 1 to round \( t \). \( \mu_t \) is the mean of data in \( X_t \) and \( n_t \) is the data size. Our approach maintains a small size sketch for the streaming data online, and we demonstrate that the hash functions can be efficiently learned based on this sketch.

Suppose we have a \( d \times n \) matrix \( X = [x_1, x_2, \cdots, x_n] \) where each column \( x_i \) is a sample in the dataset. Denote \( W = [w_1, w_2, \cdots, w_r] \in \mathbb{R}^{d \times r} \), then the objective of PCA hashing [1, 3, 7] can be formally written as:

\[
\begin{align*}
\max_{W \in \mathbb{R}^{d \times r}} & \quad tr(W^T (X - \mu)(X - \mu)^TW) \\
\text{s.t.} & \quad W^TW = I_r
\end{align*}
\]

where \( \mu = \frac{1}{n} \sum_{i=1}^n x_i \) is the mean of all the data. The notation \( (X - \mu) \) means the matrix \( [x_1 - \mu, x_2 - \mu, \cdots, x_n - \mu] \), which is equivalent to centering the data. However, this is obviously a batch based learning method and suffers from the two limitations we mentioned above.

A sketch of matrix \( P \) is another matrix \( Q \) which is much smaller than \( P \), but still preserves the properties of interest. In this way, the storage of the matrix \( Q \) will be much easier, and the computations of the sketch will be more efficient than with the original \( P \) [4, 5]. Given a matrix \( P \in \mathbb{R}^{d \times n} \), we aim to maintain a much smaller matrix \( Q \in \mathbb{R}^{d \times k} \) with \( l \ll n \) as an approximation to \( P \). The goal is to track an \( \varepsilon \)-approximation to the norm of matrix along any direction, i.e., \( \|P^T x\|_2^2 - \|Q^T x\|_2^2 \leq \varepsilon \|P\|_F^2 \). \( \forall x, \|x\|_2 = 1 \). The latest significant effort is represented by Frequent Directions (FD) proposed by Liberty [5]. Inspired by the works in finding frequent items, Liberty investigated how to apply the Misra-Gries technique [6] to matrix sketching. FD provides a tight bounds for its performance. Formally, we have

\[
0 \leq \|PP^T - QQ^T\|_2 \leq 2 \frac{\|P\|_F^2}{\|Q\|_F^2}.
\]

We attempt to employ the favorable property that \( PP^T \approx QQ^T \) in [5] to handle the scalability and streaming data issue in hashing. A very straightforward way is sketching the matrix \( X - \mu \) in Eq.(5) with [5] so that we can get a significantly smaller sketch \( Y \) which approximates \( X - \mu \) well with \( YY^T \approx (X - \mu)(X - \mu)^T \). However, it is infeasible because the data is continuously changing, and therefore the mean of data \( \mu \) changes too. When a new data chunk \( D_t \) arrives at round \( t \), since the mean of the dataset changes to \( \mu_t \), we need to re-sketch all the data \([D_1 - \mu_1, D_2 - \mu_2, \cdots, D_m - \mu_t]\). In order to avoid the mean-varying problem, one can augment every data chunk with a virtual sample, which is carefully chosen to correct the time-varying mean. Specifically, for a stream \( X_t \) we design a matrix \( E_t \) as \( E_t = \frac{1}{n_t} \sum_{i=1}^{n_t} x_i \cdots \frac{1}{n_t} \sum_{i=1}^{n_t} x_i \). We can verify the efficiency of our method in online setting.

\[\begin{align*}
\mu_t &= \frac{1}{n_t} \sum_{i=1}^{n_t} x_i \\
\max_{W \in \mathbb{R}^{d \times r}} & \quad tr(W^T (X - \mu)(X - \mu)^TW) \\
\text{s.t.} & \quad W^TW = I_r
\end{align*}\]

Thus, it allows us to find a sketch \( Y \) which approximates \( E_t \) well and then learn hash functions online based on this sketch. The overal accumulation time complexity is \( O(ndl + dld^2 + t^3) \), and the overall space complexity is \( O(ld + l^2 + dr) \).

We test the proposed methods on two benchmarks CIFAR-10 and GIST-1M. Fig.1 reports the most interesting results we think. In this experiment, we find a sketch of size 200 for the training data and then learn hash functions on this sketch. We find that our method achieve comparable performance to ITQ and outperforms most of state-of-the-art hashing methods. It implies that the sketch (even of size 200) can preserve sufficient information for hash functions learning. We also evaluate the training time of our method. By dividing the data into 100 chunks, the proposed method achieves about three times speed-up on both datasets than OKH [2], which verifies the efficiency of our method in online setting.