Illumination and Reflectance Spectra Separation of a Hyperspectral Image Meets Low-Rank Matrix Factorization

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A hyperspectral signal records the spectral radiance of a reflective surface, which is the compound of the illumination spectral power distribution and the surface reflectance spectra. To extract the illumination and the reflectance spectra from an observed hyperspectral signal has been a long-standing problem in photometric computer vision [3, 5, 7]. Specifically, for a diffuse surface point, its radiance d_i at the i-th spectral band recorded by a hyperspectral camera is proportional to the product of the illumination l_i and the surface reflectance r_i, that is,

$$d_i = l_i r_i, 1 \leq i \leq m, \tag{1}$$

where the proportional scalar, accounting for such factors as gain and exposure time, has been omitted, and m denotes the number of spectra bands. It is also assumed that the spectral sensitivity function of the hyperspectral camera has been precorrected to be one at all spectral bands. To separate the observed radiance spectra, one has to assume that both the illumination and the reflectance spectra are low-dimensional [3, 5, 7], since otherwise there would be more variables than constraints.

This paper addresses a more practical variant of the classical separation problem of a single spectral signal under restricted subspace illumination, namely, the illumination and reflectance spectra separation problem of a whole hyperspectral image captured under general spectral illumination, hereafter referred to as the IRSS problem (see Fig.1 for an example). We try to utilize a huge amount of spectral signals in a hyperspectral image to assist the separation, without imposing any restriction on the illumination spectra.

For a hyperspectral image with n pixels under spatially uniform illumination, the intensity value d_{ij} of the j-th pixel at the i-th spectral band reads

$$d_{ij} = l_i r_{ij}, 1 \leq i \leq m, 1 \leq j \leq n, \tag{2}$$

which can be stacked into a matrix system

$$\underbrace{\begin{bmatrix} d_{11} & \cdots & d_{1n} \\ \cdots & \cdots & \cdots \\ d_{m1} & \cdots & d_{mn} \end{bmatrix}}_{D_{m\times n}} = \underbrace{\begin{bmatrix} l_1 & & \\ & \cdots & \\ & & l_m \end{bmatrix}}_{L_{m\times m}} \underbrace{\begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \cdots & \cdots & \cdots \\ r_{m1} & \cdots & r_{mn} \end{bmatrix}}_{R_{m\times n}}. \tag{3}$$

The above system has mn constraints in the observation matrix D and $m(n+1)$ variables in the diagonal illumination matrix L and the reflectance matrix R, which means that the IRSS problem without any assumption on illumination nor reflectance is underconstrained, no matter how many signals we have.

It has been widely agreed that reflectance spectra lie in a low-dimensional linear subspace. Therefore, we introduce the subspace model of reflectance R, and rewrite eq.(3) into

$$D_{m\times n} = L_{m\times m} R_{m\times n} = L_{m\times m} B_{m\times s} C_{s\times n}, \tag{4}$$

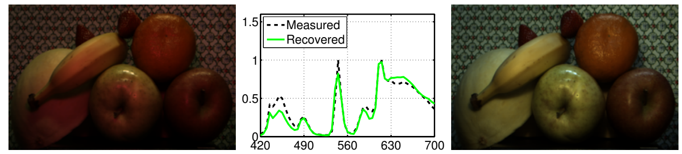
in which B and C denote the spectral bases and coefficients, respectively, and s the subspace dimensionality. According to [8, 9], s is often chosen to be around 8 so as to reach the best tradeoff between expression power and noise resistance in the process of fitting reflectance spectra.

As shown in eq.(4), the IRSS problem is actually a low-rank matrix factorization problem. More interestingly, the low-rank formulation of IRSS is very similar to that of the nonrigid structure-from-motion problem (NRSfM) [1, 2, 4, 6] in geometric vision, thus can be regarded as a spectra-domain counterpart to the NRSfM problem in time-domain.

We have proved that this IRSS problem assumes a unique solution up to an unknown scale between the illumination and reflectance components, under the standard assumption that reflectance spectra lie in a low-dimensional linear subspace. Considering that this subspace model is not perfectly errorless and the image intensity values usually suffer from noise, we also develop a scalable algorithm that works in the presence of both model error and image noise. Rather than explicitly describing the physical imaging process of those complicated effects beyond diffuse reflectance, like shadows and highlights, we treat them as outliers to our low-rank model, which can be accounted for via low-rank matrix approximation operation of a nonnegative observation matrix under the robust L_1-norm error metric.

Quantitative experiments on both synthetic data and real images have demonstrated that, our separation results of scenes with sufficient color variation are reasonably accurate, and can benefit some important applications, such as spectra relighting of a single view and illumination swapping between two different views.

Our work has left out quite a few important aspects that deserve to be explored in depth. For example, highlights, being treated as outliers to the low-rank model in this paper, are known to encode some important information of the illumination. Therefore, it would be rewarding to carefully model and exploit highlights in a physically sound way. As for the application aspect, we hope that our separation results can benefit some other potential applications, such as spectra-based material recognition and hyperspectral image compression.



(a) Input (b) Illumination (c) Reflectance

Figure 1: Given a hyperspectral image (a), the IRSS problem is to simultaneously estimate the illumination spectrum (b) and the reflectance spectra of all pixels (c). Hyperspectral images are shown in RGB for visualization.

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This is an extended abstract. The full paper is available at the Computer Vision Foundation webpage.