Abstract

It is known that time-to-contact toward objects can be estimated just from changes in the object size in camera images, and we do not need any additional information, such as camera parameters and motions. However, the existing methods for measuring the time-to-contact are based on geometric image features, such as corners and edge lines, and thus they cannot be used when there are no geometric features in images. In this paper, we propose a new method for computing the time-to-contact from photometric information in images. When a light source moves in the 3D scene, an observed intensity changes according to the motion of the light source. In this paper, we analyze the change in photometric information in images, and show that the time-to-contact can be estimated just from the changes in intensity in images. Our method does not need any additional information, such as radiance of light source, reflectance of object and orientation of object surface. The proposed method can be used in various applications, such as vehicle driver assistance.

1. Introduction

The time-to-contact is very useful for measuring the danger of collision of objects [3], and has been studied extensively [1,2,6,8,7,4,9]. It is invariant under camera parameters and structure of the scene, and thus we do not need to estimate camera parameters and scene structures for computing the time-to-contact.

The time-to-contact has been studied as differential geometric properties in images, and thus the change in apparent size of objects or the optic flow fields in images have been used for estimating the time-to-contact [10,7]. For example, Cipolla et al. [1] proposed a method for computing the time-to-contact from changes in the area of closed contours in images. However, these methods require extraction of geometric image features, such as corners, closed contours, etc. For relaxing this problem, Horn et al [4] proposed a method for estimating the time-to-contact from the brightness in images. However, their method is based on the constant brightness assumption, and thus once this assumption breaks, it cannot be used. For example, if a light source moves in the 3D scene, their method cannot be applied at all.

In this paper, we propose a method for computing the time-to-contact from image intensity. Our method is very different from the traditional methods, and it measures the time-to-contact of a light source motion. It is known that the brightness of a surface illuminated by a light source changes according to the motion of the light source. For example, the brightness of a road surface illuminated by the head lamp of a vehicle changes according to the motion of the vehicle as shown in Fig.1. The brightness becomes higher when the distance from the light source becomes smaller. Thus, the distance can be computed from the brightness, if the radiance of the light source, surface reflectance and surface orientation are known. However, it is very difficult to estimate these parameters accurately. Thus, we in this paper propose a method for estimating time-to-contact from changes in image intensity without knowing radiance of the light source, surface orientation and reflectance.

We first derive a basic method for estimating time-to-contact from image intensity. Then, we extend our method, so that we can estimate time-to-contact even if the surface is illuminated by not only a single light source but also ambient lights. Furthermore, we propose a computational method which is less sensitive to image noise, hence it is more practical.

2. Time to Contact from Geometric Information

We first consider traditional method for estimating time-to-contact from geometric information in images [1,8].

Let us consider a camera, which moves toward an object with a constant velocity. Then, the apparent size of the object in an image changes according to the distance from the object to the camera. Suppose the apparent size of the object in the image is \( y \) at time \( t \), and it changes to \( y' \) at time \( t + 1 \). If the real size of the object is \( Y \) and the distance
from the object to the camera changes from $Z$ at time $t$ to $Z'$ at time $t+1$, the image size $y$ and $y'$ can be described as follows:

$$y = \frac{fY}{Z}$$  \hspace{1cm} (1)$$
$$y' = \frac{fY'}{Z'}$$  \hspace{1cm} (2)$$

where, $f$ is the focal length of the camera. Then, the time-to-contact can be defined by using the distance $Z$ and $Z'$ as follows:

$$TC = \frac{Z'}{Z - Z'}$$  \hspace{1cm} (3)$$

From Eq.(1), Eq.(2) and Eq.(3), we find that we can estimate time-to-contact just from the size of the object in the image as follows:

$$TC = \frac{y}{y'} - y$$  \hspace{1cm} (4)$$

This equation does not include focal length, real size of the object and distance between the object and the camera, and thus we can estimate time-to-contact just from changes in geometric image information.

This method is very useful, since we do not need calibration of camera parameters, such as focal length, principal point etc. However, the method cannot be applied, if there is no geometric information in images. For example, we often cannot obtain sufficient geometric information from night images. In order to avoid the problem, we propose a novel method for estimating time-to-contact. In this method, we estimate time-to-contact not from geometric information but from photometric information.

3. Time-to-Contact from Intensity

3.1. Reflectance Model

Let us derive time-to-contact from photometric information. In order to derive a new time-to-contact based on photometric information, we first consider the photometric reflection model of observed intensity. In this paper, we consider that the surfaces are Lambertian.

Let us consider the case where a light source exists in a scene as shown in Fig.2. In this case, observed intensity $i$ can be described by a light source position $S$, surface normal $n$ and observed point $X$ as follows:

$$i = \frac{1}{||S-X||^2}E\rho \max(n^T(S - X), 0)$$  \hspace{1cm} (5)$$

where, $E$ and $\rho$ denote the radiance of the light source and the reflectance of the surface respectively. If there is no negative intensity, Eq.(5) can be rewritten as follows:

$$i = \frac{1}{||S-X||^2}E\rho n^T(S - X)$$  \hspace{1cm} (6)$$

In Eq.(6), $\frac{S-X}{||S-X||}$ describes the light source direction at $X$, and $\frac{1}{||S-X||^2}$ denotes the attenuation of light according to the distance. From Eq.(6), we find that the observed intensity is inversely proportional to the squared distance between the light source and the observed point, and the observed intensity strongly depends on the distance from the light source.

By using this property, Liao et al.\cite{5} proposed a method for estimating 3D shape from observed intensities. Unlike their method, we in this paper propose a method for estimating time-to-contact from observed intensities without recovering 3D shape explicitly. In the following sections, we analyze the intensity model and derive a method for estimating the time-to-contact of a light moving toward an object.

3.2. Time-to-Contact from Intensity

In this section, we propose a method for estimating time-to-contact from image intensity. We first consider the case where the light source moves toward the observation point $X$ as shown in Fig.3(a). In this case, the relative angle between the surface normal $n$ and the orientation of the light source $\frac{S-X}{||S-X||}$ is constant during the light source motion. We next generalize the problem and consider the case where the light source does not move toward the observation point $X$ as shown in Fig.3(b). In this case, the relative angle between the surface normal $n$ and the orientation of the light source $\frac{S-X}{||S-X||}$ changes according to the light source motion.
and camera parameters.

such as reflectance of surface, radiance of the light source

image intensity without using any other prior knowledge,

using the method, we can estimate time-to-contact just from

direction of observation point

contact in the case where the light source moves not in the

direction of the observed point.

3.3. Time-to-Contact under Simple Case

We first consider a method for computing time-to-contact from image intensities in the case where the light source moves toward the observation point X as shown in Fig. 3(a). In this case, the angle between a surface normal n and the orientation of the light source S−X is constant, and Eq. (6) can be rewritten as follows:

\[ i = \frac{k}{||S - X||^2} = \frac{k}{d^2} \]  

where, \( d \) denotes the distance between X and S, and \( k \) is a constant, since it depends only on the reflectance of surface and the radiance of a light source. Thus, square root of intensity can be computed as follows:

\[ \sqrt{i} = \frac{k'}{||S - X||} = \frac{k'}{d} \]  

where \( k' = \sqrt{k} \). Since \( \sqrt{i} \) is inversely proportional to distance \( d \) as \( y \) in Eq. (1), we can estimate time-to-contact \( TC \) from image intensity as follows:

\[ TC = \frac{\sqrt{i}}{\sqrt{i'} - \sqrt{i}} \]  

where \( i' \) indicates the observed intensity at time \( t + 1 \). By using the method, we can estimate time-to-contact just from image intensity without using any other prior knowledge, such as reflectance of surface, radiance of the light source and camera parameters.

3.4. Time-to-Contact under Complex Case

We next consider a method for computing time-to-contact in the case where the light source moves not in the direction of observation point X as shown in Fig. 3(b).

In this case, not only the distance but also the angle between the surface normal and the light source orientation changes according to the motion of the light source. For analyzing the change in angle, we rewrite the distance \( ||S - X|| \) between the light source S and the observed point X by using its horizontal distance \( d \) and vertical distance \( h \) as follows:

\[ ||S - X|| = \sqrt{d^2 + h^2} \]  

When the surface normal n is perpendicular to the moving direction of light source S, Eq. (6) can be rewritten by using the horizontal distance \( d \) and the vertical distance \( h \) as follows:

\[ i = \frac{E \rho h}{(d^2 + h^2)^{\frac{3}{2}}} \]  

Since the light source moves only in the horizontal direction, the vertical distance \( h \) is constant. Thus, by taking the derivative of the intensity \( i \) with respect to the time \( t \), we have:

\[ j = \frac{di}{dt} = \frac{di}{dd} \frac{dd}{dt} = \frac{-3sE \rho h d}{(d^2 + h^2)^{\frac{5}{2}}} \]  

where \( s = \frac{dd}{dt} \) is a speed of the light source, and we assume that it is constant. Note that, the denominator of the derivative is an exponentiation of the denominator of intensity \( i \) shown in Eq. (1). Thus, we can eliminate the term \( (d^2 + h^2) \) by computing I as follows:

\[ I = \frac{i^2}{j} = \frac{(E \rho h)^{\frac{3}{2}}}{3s} \cdot \frac{1}{d} \]  

\[ = \frac{k''}{d} \]  

where \( k'' \) is a constant, since it depends only on radiance \( E \), reflectance \( \rho \), vertical distance \( h \) and the speed of light source \( s \). Finally, we can estimate time-to-contact by using I as follows:

\[ TC = \frac{I}{I' - I} \]  

where \( I' \) is computed from Eq. (13) at time \( t + 1 \). By using the method, we can estimate time-to-contact of a moving light, even if the light source does not move toward the object.

4. Time-to-Contact from Intensity under Ambient Light

Up to now we proposed a method for estimating time-to-contact from image intensity. However, the proposed method assumes that there is only a single moving light source in the scene. Thus, if we have ambient light as well as the moving light source, the proposed method does not
work properly. In order to solve the problem, we next extend our method, so that we can estimate time-to-contact from image intensity, even if the ambient light exists in the scene.

4.1. Under Simple Case

We first consider a method for estimating time-to-contact under a simple case, where the light source moves toward the observation point as shown in Fig.3(a). When we have ambient light in the scene, the observed intensity can be described as follows:

\[ i_a = \frac{k}{\|S - X\|^2} + a = \frac{k}{d^2} + a \tag{15} \]

where \( a \) is an image intensity caused by the ambient light. Now, we take the derivative \( j_a \) of \( i_a \) with respect to the time \( t \) as follows:

\[ j_a = \frac{di_a}{dt} = \frac{-2ks}{d^3} \tag{16} \]

As shown in Eq.(16), the constant \( a \) disappears in \( j_a \), and the cube root of \( j_a \) is inversely proportional to the distance \( d \). Thus, time-to-contact \( TC \) can be computed as follows:

\[ TC = \frac{\sqrt[3]{j_a}}{\sqrt[3]{j_a} - \sqrt[3]{j_a}} \tag{17} \]

In this equation, the effect of ambient light \( a \) is eliminated completely, and thus we can estimate time-to-contact even if there exists ambient light in the scene.

4.2. Under Complex Case

We next consider the case where the light source moves not in the direction of observation point \( X \) as shown in Fig.3(b). From Eq.(11), we find that the observed intensity \( i_a \) with the ambient light component \( a \) can be described as follows:

\[ i_a = \frac{E \rho h}{(d^2 + h^2)^{3/2}} + a \tag{18} \]

Again, we compute the derivative of \( i_a \) with respect to the time \( t \) as follows:

\[ \frac{di_a}{dt} = \frac{-3sE \rho hd}{(d^2 + h^2)^{5/2}} \tag{19} \]

Since the ambient term \( a \) is constant, it disappears in Eq.(19). Now, we assume that the height \( h \) of light source is sufficiently small comparing with the distance \( d \), and thus, \( h^2 \) in Eq.(19) can be ignored. Then, Eq.(19) can approximately be described as follows:

\[ \frac{di_a}{dt} \approx \frac{-3sE \rho hd}{(d^2)^{5/2}} = \frac{k}{d^3} = j_a \tag{20} \]

Since the fourth root of \( j_a \) is inversely proportional to the distance \( d \), the time-to-contact \( TC \) can be estimated approximately as follows:

\[ TC = \frac{\sqrt[3]{j_a}}{\sqrt[3]{j_a} - \sqrt[3]{j_a}} \tag{21} \]

Since the ambient term disappears in Eq.(21), we can estimate the time-to-contact even if there exists ambient light in the scene.

5. Time-to-Contact without using Image Derivatives

5.1. Under Complex Case

So far we proposed a method for estimating time-to-contact by using derivatives of image intensities. The proposed method is useful when we obtain image intensity accurately. However, observed images often include image noises, and thus, we may not be able to compute image derivatives reliably because of the image noises. In particular, when the distance from a light source to an observed point is large, the image noise becomes relatively large comparing with the image derivatives. Thus, in this section, we propose an efficient method for estimating time-to-contact without using image derivatives. Although the proposed method is an approximation, it is more stable than the method proposed in the previous section.

Let us consider an intensity \( i' \) at time \( t + 1 \). Since the intensity \( i \) at time \( t \) is as shown in Eq.(18), the intensity \( i' \) at time \( t + 1 \) can be described as follows:

\[ i' = \frac{E \rho h}{((d + s)^2 + h^2)^{3/2}} \tag{22} \]

where \( s \) is a light source motion in a unit time. Then, by taking the \( \frac{3}{2} \) power of \( i \) and \( i' \), we have:

\[ i^{\frac{3}{2}} = \frac{(E \rho h)^{3/2}}{d^3 + h^2} \tag{23} \]

\[ i'^{\frac{3}{2}} = \frac{(E \rho h)^{3/2}}{(d + s)^2 + h^2} \tag{24} \]

From Eq.(23) and Eq.(24), we find that the relationship between \( i^{\frac{3}{2}} \) and \( i'^{\frac{3}{2}} \) can be described by using \( d, s \) and \( h \) as follows:

\[ \frac{i'^{\frac{3}{2}} - i^{\frac{3}{2}}}{i'^{\frac{3}{2}} + i^{\frac{3}{2}}} = \frac{d^2 + h^2}{2ds + s^2} \tag{25} \]

In general light source motion \( s \) is sufficiently small comparing to the distance \( d \). Also, vertical distance \( h \) is often very small comparing to the horizontal distance \( d \). Thus, the term \( s^2 \) and \( h^2 \) in Eq.(25) are negligible, assuming \( s \) and \( h \)
are sufficiently smaller than \( d \). As a result, Eq. (25) can be approximated as follows:

\[
\frac{\sqrt[2]{i}}{\sqrt[2]{i} - \sqrt[2]{i'}} \sim \frac{d}{2s} \tag{26}
\]

Since the time-to-contact \( TC \) can be defined by \( \frac{d}{2s} \), we can estimate the time-to-contact \( TC \) approximately by using \( i \) and \( i' \) as follows:

\[
TC \sim \frac{2\sqrt[2]{i'}}{\sqrt[2]{i} - \sqrt[2]{i'}} \tag{27}
\]

In this equation, we do not need derivatives of intensities unlike Eq. (14). Thus, the stability of time-to-contact computed from Eq. (27) is much higher than that computed from Eq. (14).

### 5.2. Simultaneous Estimation of Ambient Light and Time-to-Contact

Although we derived a method for computing time-to-contact without using image derivatives, it is limited to the case where the ambient light does not exist. To cope with the ambient light problem without using image derivatives, we in this section propose a method for estimating time-to-contact and ambient light simultaneously.

We first consider the case when a light source moves toward observed point as shown in Fig. 3 (a). Suppose \( a \) is an ambient light term. Then from Eq. (9), we find that the time-to-contact \( TC \) can be estimated as follows:

\[
TC = \frac{\sqrt{i} - a}{\sqrt{i'} - a - \sqrt{i} - a} \tag{28}
\]

where \( i \) and \( i' \) indicate observed intensities at time \( t \) and \( t + 1 \) respectively. Since the light source moves with a constant speed, the time-to-contact in a previous time can be estimated as follows:

\[
TC - 1 = \frac{\sqrt{i''} - a}{\sqrt{i''} - a - \sqrt{i'} - a} \tag{29}
\]

where \( i'' \) is an intensity at \( t + 2 \). From Eq. (28) and Eq. (29), the relationship between the ambient term \( a \) and the observed intensities can be described as follows:

\[
\frac{\sqrt{i} - a}{\sqrt{i'} - a - \sqrt{i} - a} = \frac{\sqrt{i''} - a}{\sqrt{i''} - a - \sqrt{i'} - a} + 1 \tag{30}
\]

Thus, we can estimate the ambient term \( a \) by minimizing the following equation:

\[
e = \frac{\sqrt{i} - a}{\sqrt{i'} - a - \sqrt{i} - a} - \frac{\sqrt{i''} - a}{\sqrt{i''} - a - \sqrt{i'} - a} - 1 \tag{31}
\]

By using the estimated \( a \), we can compute the time-to-contact by using Eq. (28), even if there exists ambient light in the scene.

We next consider the complex case shown in Fig. 3 (b). Following Eq. (30), the time-to-contact in this case can be estimated as follows:

\[
TC = \frac{2(i - a)^{\frac{1}{2}}}{(i' - a)^{\frac{1}{2}} - (i - a)^{\frac{1}{2}}} \tag{32}
\]

From the relationship between the ambient term \( a \) and the intensities, we define an evaluation value for the ambient term as follows:

\[
e = \frac{2(i - a)^{\frac{1}{2}}}{(i' - a)^{\frac{1}{2}} - (i - a)^{\frac{1}{2}}} - \frac{2(i' - a)^{\frac{1}{2}}}{(i'' - a)^{\frac{1}{2}} - (i' - a)^{\frac{1}{2}}} - 1 \tag{33}
\]

By minimizing \( e \), we can estimate the ambient term \( a \), and thus we can estimate the time-to-contact under complex case by using Eq. (32).

In practice, these techniques are more robust against image noise than the aforementioned methods which use image derivatives, since the effect of image noise is larger in the derivative of intensities than in intensities themselves.

### 6. Experimental Results

#### 6.1. In the Case of a Facing Target

In this section, we show experimental results from the proposed method. We first show experimental results when the light source faces to the object. Fig. 4 (c) shows the experimental setup. In this experiment, the light source shown in Fig. 4 (a) was used and a camera observed a plaster cube shown in Fig. 4 (b). The light source illuminated the plaster cube as shown in Fig. 4 (c), and moved horizontally toward the plaster cube. The distance from the light to the object changes from 200 cm to 50 cm. The moving speed of the light source was 10 cm/sec. The images were taken by the fixed camera every second. There were some other static light sources as the ambient light in the scene. The examples of observed images are shown in Fig. 5. Intensities used for estimating the time-to-contact were obtained by averaging the intensity in 100 pixels \( \times \) 100 pixels at the image center. From these images the time-to-contact was estimated by using three methods, which are (a) the method shown in Eq. (9) which does not consider ambient light, (b) the method shown in Eq. (17) which eliminates ambient light using image derivatives, and (c) the method shown in Eq. (28) which eliminates ambient light without using image derivatives.

The estimated time-to-contact is shown in Fig. 6. The result from (a) shown by the blue line has large off-set from the ground truth shown by the black line. This is because the method (a) does not count the ambient light which exists in the image. The result from (b) shown by the green line is more accurate than that of (a), when the distance from the light to the observed point is small. However, the results
become unstable when the distance becomes large. This is because image derivatives used in this method become unstable when the change in intensity becomes small in large distance. The result from (c) shown by the red line is the best in these three methods as shown in Fig.6. It is more stable than (b), and is more accurate than (a). From these results, we find that we can eliminate the effect of ambient light appropriately, and can estimate the time-to-contact accurately by using the method (c).

6.2. In the Case of a Non-Facing Target

We next show experimental results in the case where the observed point is not in the direction of the light source motion. Fig.7 shows the experimental environment.

In this experiment, the top surface of the object was illuminated by the light source, and it was observed by the fixed camera. This experiment corresponds to the case of Fig.3(b). The light source moved along with the blue arrow changing the distance from 200 cm to 50 cm. The examples of observed images are shown in Fig.8. In this experiment, camera parameters for controlling image brightness were changed every 3 seconds, so that we obtain sufficiently bright images. Although we do not have any information on the brightness parameters of the camera, the set of two images for estimating time-to-contact were taken under the same camera parameters, and thus there is no problem for estimating time-to-contact from the proposed method.

The time-to-contacts were estimated by using three methods, that is (a) the method shown in Eq.(14) which does not eliminate ambient light, (b) the method shown in Eq.(21) which eliminates ambient light using image derivatives, and (c) the method shown in Eq.(32) which eliminates ambient light without using image derivatives. The average intensity of 200 pixels × 200 pixels at the image center was used for estimating the time-to-contact. The derivatives of intensity for method (a) and (b) were computed numerically by using images taken at two consecutive instants. The speed of the light source motion was 10 cm/sec, and images were taken every second. The horizontal distance between the light source and the observed object was 50 cm.

The Estimated time-to-contact is shown in Fig.9. In this result, the method (c) which has ambient elimination provides the best result. Although the method (a) often provides better result, it is not stable since it is noise sensi-
Figure 9. Estimated time to contact. The time-to-contact was estimated by using three methods. The black line indicates the ground truth, the blue line indicates results without ambient light elimination, the green line indicates results from image derivative and the red line indicates results with ambient light elimination.

tive. Although the method (b) provides us stable result, it includes effect of the ambient light. These results indicates that the time-to-contact estimated from method (c) is stable and accurate.

6.3. In Real Environment

Finally, we show experimental results from the tail light of a vehicle in an outdoor scene as shown in Fig.10. In this experiment, the vehicle moved backward toward the wall in Fig.10. The wall was illuminated by the tail lump of the vehicle, and was observed by the camera. As shown in this figure, there is no remarkable geometric feature on the wall. The speed of the vehicle was 1 m/sec. The camera captured images every second, and the time-to-contact at each time instant was estimated from images by using method (a), (b) and (c) as in the previous experiment. Fig.11 (a) and (b) show a pair of consecutive images obtained from the camera and used for computing the time-to-contact. Fig.11 (c) and (d) show another pair of consecutive images. As shown in these images, there is no geometric feature such as edges and corners in images, and thus we cannot estimate time-to-contact by using the existing methods based on geometric information. However our proposed method can estimate the time-to-contact from the changes in image intensity.

The estimated time-to-contact from these images are shown in Fig.12. In this figure, the blue line shows the result from method (a), the green line shows the result from method (b), the red line shows the result from method (c), while the black line shows the ground truth. The estimated time-to-contact from method (c) is very close to the ground truth, while the result from method (a) has some off-set caused by the ambient light and the result from method (b) is noise sensitive. From these results, we find that the proposed method can estimate time-to-contact accurately, even if there is no remarkable image feature, and even if there exist ambient lights in the scene.

We also show results from the head light of a vehicle as shown in Fig.13. In this experiment, the vehicle moved toward the wall. The light emitted from the head lump is not exactly a point light source. Also, the reflectance property of the wall surface is not exactly Lambertian. However, we still tried our method, and evaluated its applicability toward these situations. Fig.14 shows some examples of input images. In these images, the wall was illuminated by the head light. Note, the intensity gain of the camera was controlled for each pair of images, so that we have enough intensity in images.

The estimated time-to-contact from these images are shown in Fig.15. In this figure, the blue line shows the method which does not consider ambient light, and the red
Figure 13. Experimental environment (head light)

(a) 4m  (b) 6m  (c) 16m  (d) 18m
Figure 14. Examples of observed images.

Figure 15. Estimated time-to-contact from observed intensities. The red line shows the result from a method with ambient elimination, and the blue line shows the result from a method without ambient elimination.

line shows the method which eliminates ambient light without using image derivatives. Again, the method which eliminates ambient light without using image derivatives provides us the better result. These results show that the proposed method is robust against the variation of light sources, surface reflectance and ambient light in the scene.

7. Conclusion

In this paper, we proposed a method for computing time-to-contact by using photometric information. When a light source moves in the scene, an observed intensity changes according to the motion of the light source. In this paper, we analyzed the change in intensity in camera images, and showed that the time-to-contact can be estimated just from the change in intensity in images.

We first derived a basic method for estimating the time-to-contact of a light source from changes in image intensity. Then, we extend our method, so that we can estimate time-to-contact even if the ambient light exists in the scene. Furthermore, we showed a robust method which is less sensitive to image noises.

These proposed methods were implemented and compared their accuracy in real image experiments. We showed that ambient light estimations as well as the time-to-contact estimation is efficient in the sense of accuracy and stability under image noises.

Our method does not need any additional information, such as radiance of light source, reflectance of object and orientation of object surface. Hence, the proposed method can be used in various applications, such as vehicle driver assistance.

References